Chapter 11

Strip Lines

11-0 INTRODUCTION

Prior to 1965 nearly all microwave equipment utilized coaxial, waveguide, or parallel strip-line circuits. In recent years—with the introduction of monolithic microwave integrated circuits (MMICs)—microstrip lines and coplanar strip lines have been used extensively, because they provide one free and accessible surface on which solid-state devices can be placed. In this chapter parallel, coplanar, and shielded strip lines and microstrip lines, which are shown in Fig. 11-0-1 [1], are described.

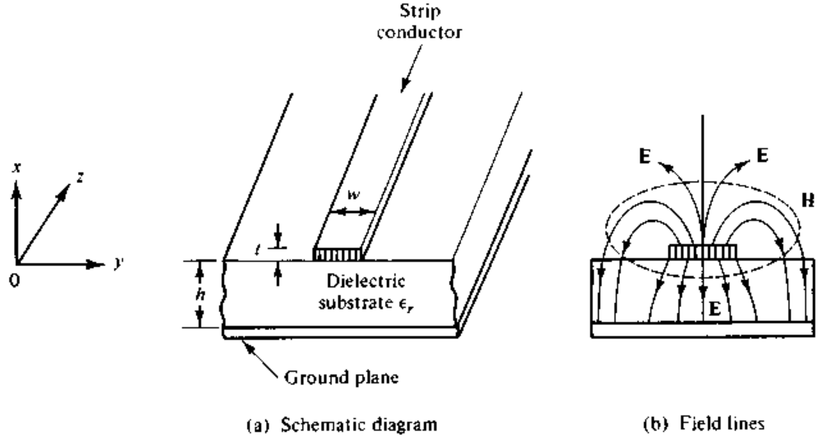

Figure 11-0-1 Schematic diagrams of strip lines.

11-1 MICROSTRIP LINES

Chapter 3 described and discussed conventional transmission lines in detail. All electrical and electronic devices with high-power output commonly use conventional lines, such as coaxial lines or waveguides, for power transmission. However, the microwave solid-state device is usually fabricated as a semiconducting chip with a volume on the order of 0.008–0.08 mm^3. The method of applying signals to the chips and extracting output power from them is entirely different from that used for vacuum-tube devices. Microwave integrated circuits with microstrip lines are commonly used with the chips.The microstrip line is also called an *open-strip line*. In engineering applications, MKS units have not been universally adopted for use in designing the microstrip line. In this section we use either English units or MKS units, depending on the application, for practical purposes.

Modes on microstrip line are only quasi-transverse electric and magnetic (TEM). Thus the theory of TEM-coupled lines applies only approximately. Radiation loss in microstrip lines is a problem, particularly at such discontinuities as short-circuit posts, corners, and so on. However, the use of thin, high-dielectric materials considerably reduces the radiation loss of the open strip. A microstrip line has an advantage over the balanced-strip line because the open strip has better interconnection features and easier fabrication. Several researchers have analyzed the circuit of a microstrip line mounted on an infinite dielectric substrate over an infinite ground plane [2 to 5]. Numerical analysis of microstrip lines, however, requires large digital computers, whereas microstrip-line problems can generally be solved by conformal transformations without requiring complete numerical calculations.

11-1-1 Characteristic Impedance of Microstrip Lines

Microstrip lines are used extensively to interconnect high-speed logic circuits in digital computers because they can be fabricated by automated techniques and they provide the required uniform signal paths. Figure 11-1-1 shows cross sections of a microstrip line and a wire-over-ground line for purposes of comparison.

In Fig. 11-1-1(a) you can see that the characteristic impedance of a microstrip

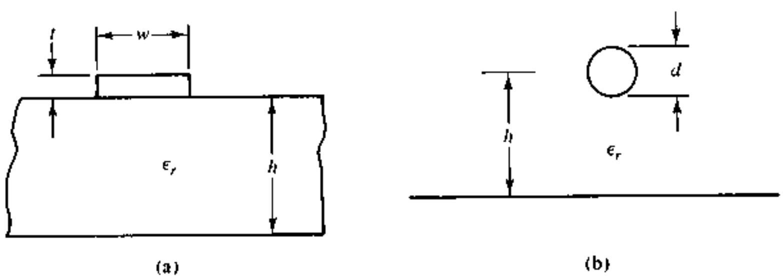

Figure 11-1-1 Cross sections of (a) a microstrip line and (b) a wire-over-ground line.

line is a function of the strip-line width, the strip-line thickness, the distance between the line and the ground plane, and the homogeneous dielectric constant of the board material. Several different methods for determining the characteristic impedance of a microstrip line have been developed. The field-equation method was employed by several authors for calculating an accurate value of the characteristic impedance [3 to 5]. However, it requires the use of a large digital computer and is extremely complicated. Another method is to derive the characteristic-impedance equation of a microstrip line from a well-known equation and make some changes [2]. This method is called a *comparative*, or an *indirect*, method. The well-known equation of the characteristic impedance of a wire-over-ground transmission line, as shown in Fig. 11-1-1(b), is given by

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln \frac{4h}{d} \qquad \text{for } h \gg d \tag{11-1-1}$$

where ϵ_r = dielectric constant of the ambient medium
h = the height from the center of the wire to the ground plane
d = diameter of the wire

If the effective or equivalent values of the relative dielectric constant ϵ_r of the ambient medium and the diameter d of the wire can be determined for the microstrip line, the characteristic impedance of the microstrip line can be calculated.

Effective dielectric constant ϵ_{re}. For a homogeneous dielectric medium, the propagation-delay time per unit length is

$$T_d = \sqrt{\mu\epsilon} \tag{11-1-2}$$

where μ is the permeability of the medium and ϵ is the permittivity of the medium. In free space, the propagation-delay time is

$$T_{df} = \sqrt{\mu_0\epsilon_0} = 3.333 \quad \text{ns/m} \quad \text{or} \quad 1.016 \quad \text{ns/ft} \tag{11-1-3}$$

where

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m, or } 3.83 \times 10^{-7} \text{ H/ft}$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m, or } 2.69 \times 10^{-12} \text{ F/ft}$$

In transmission lines used for interconnections, the relative permeability is 1. Consequently, the propagation-delay time for a line in a nonmagnetic medium is

$$T_d = 1.106\sqrt{\epsilon_r} \qquad \text{ns/ft} \tag{11-1-4}$$

The effective relative dielectric constant for a microstrip line can be related to the relative dielectric constant of the board material. DiGiacomo and his coworkers discovered an empirical equation for the effective relative dielectric constant of a microstrip line by measuring the propagation-delay time and the relative dielectric constant of several board materials, such as fiberglass-epoxy and nylon phenolic [6].

The empirical equation, as shown in Fig. 11-1-2, is expressed as

$$\epsilon_{re} = 0.475\epsilon_r + 0.67 \qquad (11\text{-}1\text{-}5)$$

where ϵ_r is the relative dielectric constant of the board material and ϵ_{re} is the effective relative dielectric constant for a microstrip line.

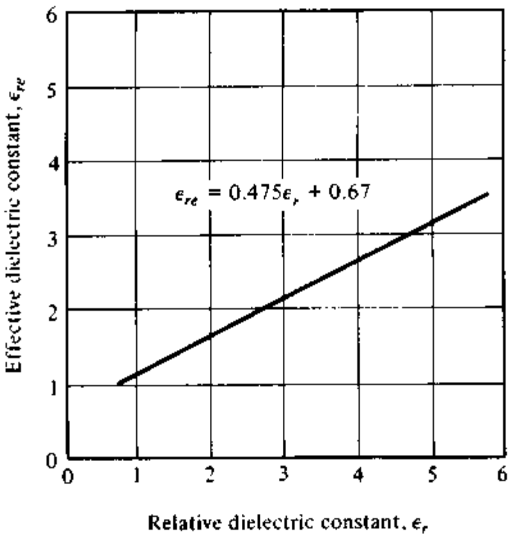

Figure 11-1-2 Effective dielectric constant as a function of relative dielectric constant for a microstrip line. (*After H. R., Kaupp [2]; reprinted by permission of IEEE, Inc.*)

Transformation of a rectangular conductor into an equivalent circular conductor. The cross-section of a microstrip line is rectangular, so the rectangular conductor must be transformed into an equivalent circular conductor. Springfield discovered an empirical equation for the transformation [7]. His equation is

$$d = 0.67w\left(0.8 + \frac{t}{w}\right) \qquad (11\text{-}1\text{-}6)$$

where d = diameter of the wire over ground
w = width of the microstrip line
t = thickness of the microstrip line

The limitation of the ratio of thickness to width is between 0.1 and 0.8, as indicated in Fig. 11-1-3.

Characteristic impedance equation. Substituting Eq. (11-1-5) for the dielectric constant and Eq. (11-1-6) for the equivalent diameter in Eq. (11-1-1) yields

$$Z_0 = \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln\left[\frac{5.98h}{0.8w + t}\right] \qquad \text{for } (h < 0.8w) \qquad (11\text{-}1\text{-}7)$$

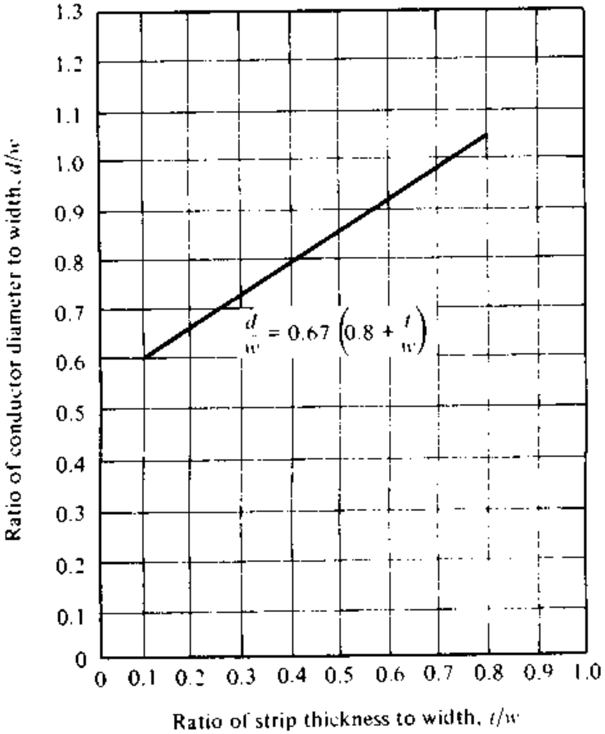

Figure 11-1-13 Relationship between a round conductor and a rectangular conductor far from its ground plane. (*After H. R. Kaupp [2]; reprinted by permission of IEEE, Inc.*)

where ϵ_r = relative dielectric constant of the board material
h = height from the microstrip line to the ground
w = width of the microstrip line
t = thickness of the microstrip line

Equation (11-1-7) is the equation of characteristic impedance for a narrow microstrip line. The velocity of propagation is

$$v = \frac{c}{\sqrt{\epsilon_{re}}} = \frac{3 \times 10^8}{\sqrt{\epsilon_{re}}} \qquad \text{m/s} \tag{11-1-8}$$

The characteristic impedance for a wide microstrip line was derived by Assadourian and others [8] and is expressed by

$$Z_0 = \frac{h}{w}\sqrt{\frac{\mu}{\epsilon}} = \frac{377}{\sqrt{\epsilon_r}}\frac{h}{w} \qquad \text{for } (w \gg h) \tag{11-1-9}$$

Limitations of Equation (11-1-7). Most microstrip lines are made from boards of copper with a thickness of 1.4 or 2.8 mils (1 or 2 ounces of copper per square foot). The narrowest widths of lines in production are about 0.005–0.010 in. Line widths are usually less than 0.020 in.; consequently, ratios of thickness to width of less than 0.1 are uncommon. The straight-line approximation from Eq. (11-1-6) is an accurate value of characteristic impedance, or the ratio of thickness to width between 0.1 and 0.8.

Since the dielectric constant of the materials used does not vary excessively with frequency, the dielectric constant of a microstrip line can be considered independent of frequency. The validity of Eq. (11-1-7) is doubtful for values of dielectric thickness h that are greater than 80% of the line width w. Typical values for the characteristic impedance of a microstrip line vary from 50 Ω to 150 Ω, if the values of the parameters vary from $\epsilon_r = 5.23$, $t = 2.8$ mils, $w = 10$ mils, and $h = 8$ mils to $\epsilon_r = 2.9$, $t = 2.8$ mils, $w = 10$ mils, and $h = 67$ mils [2].

Example 11-1-1: Characteristic Impedance of Microstrip Line

A certain microstrip line has the following parameters:

$$\epsilon_r = 5.23$$
$$h = 7 \text{ mils}$$
$$t = 2.8 \text{ mils}$$
$$w = 10 \text{ mils}$$

Calculate the characteristic impedance Z_0 of the line.

Solution

$$Z_0 = \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln\left[\frac{5.98h}{0.8w + t}\right]$$
$$= \frac{87}{\sqrt{5.23 + 1.41}} \ln\left[\frac{5.98 \times 7}{0.8 \times 10 + 2.8}\right]$$
$$= 45.78\ \Omega$$

11-1-2 Losses in Microstrip Lines

Microstrip transmission lines consisting of a conductive ribbon attached to a dielectric sheet with conductive backing (see Fig. 11-1-4) are widely used in both microwave and computer technology. Because such lines are easily fabricated by printed-circuit manufacturing techniques, they have economic and technical merit.

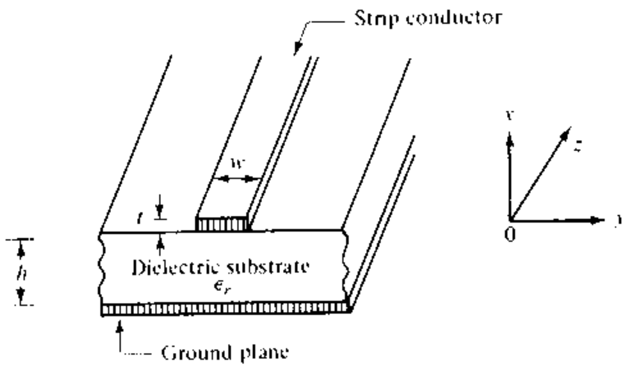

Figure 11-1-4 Schematic diagram of a microstrip line.

The characteristic impedance and wave-propagation velocity of a microstrip line was analyzed in Section 11-1-1. The other characteristic of the microstrip line is its attenuation. The attenuation constant of the dominant microstrip mode depends on geometric factors, electrical properties of the substrate and conductors, and on the frequency. For a nonmagnetic dielectric substrate, two types of losses occur in the dominant microstrip mode: (1) dielectric loss in the substrate and (2) ohmic skin loss in the strip conductor and the ground plane. The sum of these two losses may be expressed as losses per unit length in terms of an attenuation factor α. From ordinary transmission-line theory, the power carried by a wave traveling in the positive z direction is given by

$$P = \tfrac{1}{2} VI^* = \tfrac{1}{2}(V_+ e^{-\alpha z} I_+ e^{-\alpha z}) = \tfrac{1}{2} \frac{|V_+|^2}{Z_0} e^{-2\alpha z} = P_0 e^{-2\alpha z} \qquad \text{(11-1-10)}$$

where $P_0 = |V_+|^2/(2Z_0)$ is the power at $z = 0$.

The attenuation constant α can be expressed as

$$\alpha = -\frac{dP/dz}{2P(z)} = \alpha_d + \alpha_c \qquad \text{(11-1-11)}$$

where α_d is the dielectric attenuation constant and α_c is the ohmic attenuation constant.

The gradient of power in the z direction in Eq. (11-1-11) can be further expressed in terms of the power loss per unit length dissipated by the resistance and the power loss per unit length in the dielectric. That is,

$$\begin{aligned} -\frac{dP(z)}{dz} &= -\frac{d}{dz}(\tfrac{1}{2} VI^*) \\ &= \tfrac{1}{2}\left(-\frac{dV}{dz}\right) I^* + \tfrac{1}{2}\left(-\frac{dI^*}{dz}\right) V \\ &= \tfrac{1}{2}(RI)I^* + \tfrac{1}{2}\sigma V^* V \\ &= \tfrac{1}{2}|I|^2 R + \tfrac{1}{2}|V|^2 \sigma = P_c + P_d \end{aligned} \qquad \text{(11-1-12)}$$

where σ is the conductivity of the dielectric substrate board.

Substitution of Eq. (11-1-12) into Eq. (11-1-11) results in

$$\alpha_d \simeq \frac{P_d}{2P(z)} \qquad \text{Np/cm} \qquad \text{(11-1-13)}$$

and

$$\alpha_c \simeq \frac{P_c}{2P(z)} \qquad \text{Np/cm} \qquad \text{(11-1-14)}$$

Dielectric losses. As stated in Section 2-5-3, when the conductivity of a dielectric cannot be neglected, the electric and magnetic fields in the dielectric are no longer in time phase. In that case the dielectric attenuation constant, as expressed in Eq. (2-5-20), is given by

$$\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} \qquad \text{Np/cm} \tag{11-1-15}$$

where σ is the conductivity of the dielectric substrate board in ℧/cm. This dielectric constant can be expressed in terms of dielectric loss tangent as shown in Eq. (2-5-17):

$$\tan\theta = \frac{\sigma}{\omega\epsilon} \tag{11-1-16}$$

Then the dielectric attenuation constant is expressed by

$$\alpha_d = \frac{\omega}{2}\sqrt{\mu\epsilon}\tan\theta \qquad \text{Np/cm} \tag{11-1-17}$$

Since the microstrip line is a nonmagnetic mixed dielectric system, the upper dielectric above the microstrip ribbon is air, in which no loss occurs. Welch and Pratt [9] derived an expression for the attenuation constant of a dielectric substrate. Later on, Pucel and his coworkers [10] modified Welch's equation [9]. The result is

$$\alpha_d = 4.34\frac{q\sigma}{\sqrt{\epsilon_{re}}}\sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$= 1.634 \times 10^3 \frac{q\sigma}{\sqrt{\epsilon_{re}}} \qquad \text{dB/cm} \tag{11-1-18}$$

In Eq. (11-1-18) the conversion factor of 1 Np = 8.686 dB is used, ϵ_{re} is the effective dielectric constant of the substrate, as expressed in Eq. (11-1-5), and q denotes the dielectric filling factor, defined by Wheeler [3] as

$$q = \frac{\epsilon_{re} - 1}{\epsilon_r - 1} \tag{11-1-19}$$

We usually express the attenuation constant per wavelength as

$$\alpha_d = 27.3\left(\frac{q\epsilon_r}{\epsilon_{re}}\right)\frac{\tan\theta}{\lambda_g} \qquad \text{dB}/\lambda_g \tag{11-1-20}$$

where $\lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_{re}}}$ and λ_0 is the wavelength in free space, or

$\lambda_g = \frac{c}{f\sqrt{\epsilon_{re}}}$ and c is the velocity of light in vacuum.

If the loss tangent, $\tan\theta$, is independent of frequency, the dielectric attenuation per wavelength is also independent of frequency. Moreover, if the substrate conductivity is independent of frequency, as for a semiconductor, the dielectric attenuation per unit is also independent of frequency. Since q is a function of ϵ_r and w/h, the filling factors for the loss tangent $q\epsilon_n/\epsilon_{re}$ and for the conductivity $q/\sqrt{\epsilon_{re}}$ are also functions of these quantities. Figure 11-1-5 shows the loss-tangent filling factor against w/h for a range of dielectric constants suitable for microwave inte-

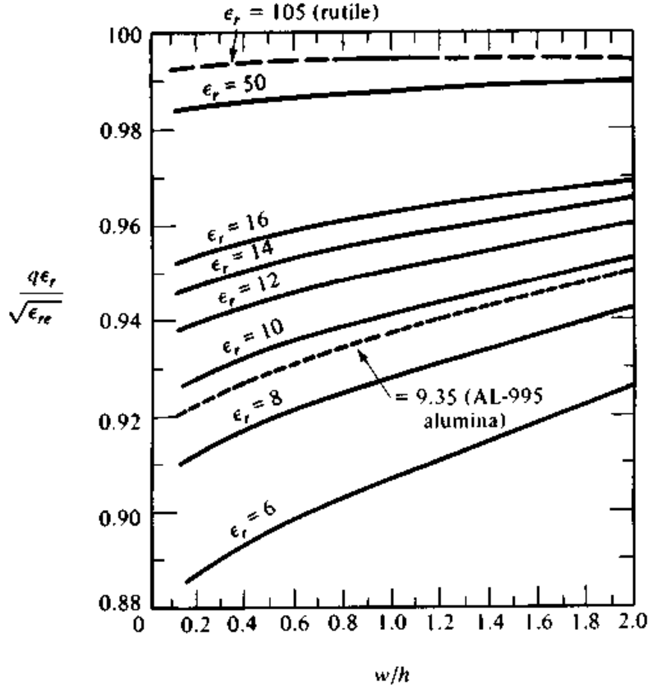

Figure 11-1-5 Filling factor for loss tangent of microstrip substrate as a function of w/h. *(After R. A. Pucel et al. [10]; reprinted by permission of IEEE, Inc.)*

grated circuits. For most practical purposes, this factor is considered to be 1. Figure 11-1-6 illustrates the product $\alpha_d \rho$ against w/h for two semiconducting substrates, silicon and gallium arsenide, that are used for integrated microwave circuits. For design purposes, the conductivity filling factor, which exhibits only a mild dependence on w/h, can be ignored.

Ohmic losses. In a microstrip line over a low-loss dielectric substrate, the predominant sources of losses at microwave frequencies are the nonperfect conductors. The current density in the conductors of a microstrip line is concentrated in a sheet that is approximately a skin depth thick inside the conductor surface and exposed to the electric field. Both the strip conductor thickness and the ground plane thickness are assumed to be at least three or four skin depths thick. The current density in the strip conductor and the ground conductor is not uniform in the transverse plane. The microstrip conductor contributes the major part of the ohmic loss. A diagram of the current density J for a microstrip line is shown in Fig. 11-1-7.

Because of mathematical complexity, exact expressions for the current density of a microstrip line with nonzero thickness have never been derived [10]. Several researchers [8] have assumed, for simplicity, that the current distribution is uniform and equal to I/w in both conductors and confined to the region $|x| < w/2$. With this assumption, the conducting attenuation constant of a wide microstrip line is given by

$$\alpha_c \simeq \frac{8.686 R_s}{Z_0 w} \qquad \text{dB/cm} \quad \text{for } \frac{w}{h} > 1 \tag{11-1-21}$$

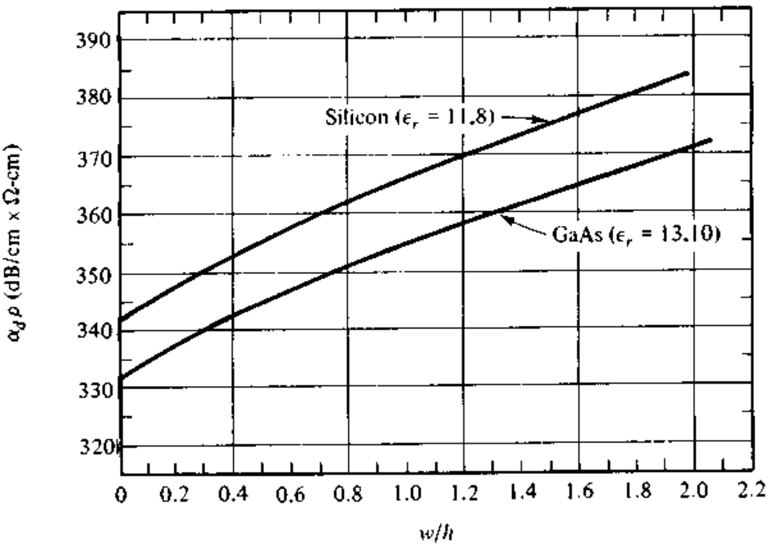

Figure 11-1-6 Dielectric attenuation factor of microstrip as a function of w/h for silicon and gallium arsenide substrates. (*After R. A. Pucel et al. [10]; reprinted by permission of IEEE, Inc.*)

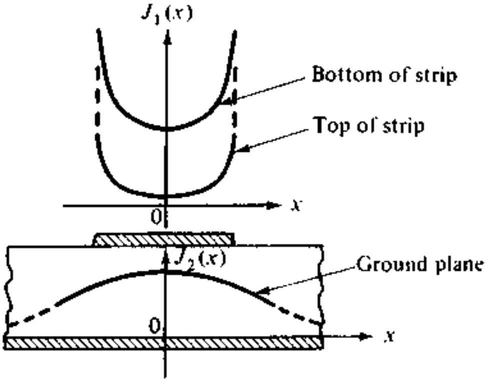

Figure 11-1-7 Current distribution on microstrip conductors. (*After R. A. Pucel et al. [10]; reprinted by permission of IEEE, Inc.*)

where $R_s = \sqrt{\dfrac{\pi f \mu}{\sigma}}$ is the surface skin resistance in Ω/square,

$R_s = \dfrac{1}{\delta\sigma}$ is Ω/square

$\delta = \sqrt{\dfrac{1}{\pi f \mu \sigma}}$ is the skin depth in cm

For a narrow microstrip line with $w/h < 1$, however, Eq. (11-1-21) is not applicable. The reason is that the current distribution in the conductor is not uniform, as assumed. Pucel and his coworkers [10, 11] derived the following three formulas from the results of Wheeler's work [3]:

$$\frac{\alpha_c Z_0 h}{R_s} = \frac{8.68}{2\pi}\left[1 - \left(\frac{w'}{4h}\right)^2\right]\left[1 + \frac{h}{w'} + \frac{h}{\pi w'}\left(\ln\frac{4\pi w}{t} + \frac{t}{w}\right)\right]$$

$$\text{for } \frac{w}{h} \le \frac{1}{2\pi} \quad (11\text{-}1\text{-}22)$$

$$\frac{\alpha_c Z_0 h}{R_s} = \frac{8.68}{2\pi}\left[1 - \left(\frac{w'}{4h}\right)^2\right]\left[1 + \frac{h}{w'} + \frac{h}{w'}\left(\ln\frac{2h}{t} - \frac{t}{h}\right)\right]$$

$$\text{for } \frac{1}{2\pi} < \frac{w}{h} \le 2 \quad (11\text{-}1\text{-}23)$$

and

$$\frac{\alpha_c Z_0 h}{R_s} = \frac{8.68}{\left\{\frac{w'}{h} + \frac{2}{\pi}\ln\left[2\pi e\left(\frac{w'}{2h} + 0.94\right)\right]\right\}^2}\left[\frac{w'}{h} + \frac{w'/(\pi h)}{\frac{w'}{2h} + 0.94}\right]$$

$$\times\left[1 + \frac{h}{w'} + \frac{h}{\pi w'}\left(\ln\frac{2h}{t} - \frac{t}{h}\right)\right] \qquad \text{for } 2 \le \frac{w}{h} \quad (11\text{-}1\text{-}24)$$

where α_c is expressed in dB/cm and

$$e = 2.718$$

$$w' = w + \Delta w \quad (11\text{-}1\text{-}25)$$

$$\Delta w = \frac{t}{\pi}\left(\ln\frac{4\pi w}{t} + 1\right) \qquad \text{for } \frac{2t}{h} < \frac{w}{h} \le \frac{\pi}{2} \quad (11\text{-}1\text{-}26)$$

$$\Delta w = \frac{t}{\pi}\left(\ln\frac{2h}{t} + 1\right) \qquad \text{for } \frac{w}{h} \ge \frac{\pi}{2} \quad (11\text{-}1\text{-}27)$$

The values of α_c obtained from solving Eqs. (11-1-22) through (11-1-24) are plotted in Fig. 11-1-8. For purposes of comparison, values of α_c based on Assadourian and Rimai's Eq. (11-1-21) are also shown.

Radiation losses. In addition to the conductor and dielectric losses, microstrip line also has radiation losses. The radiation loss depends on the substrate's thickness and dielectric constant, as well as its geometry. Lewin [12] has calculated the radiation loss for several discontinuities using the following approximations:

1. TEM transmission
2. Uniform dielectric in the neighborhood of the strip, equal in magnitude to an effective value
3. Neglect of radiation from the transverse electric (TE) field component parallel to the strip
4. Substrate thickness much less than the free-space wavelength

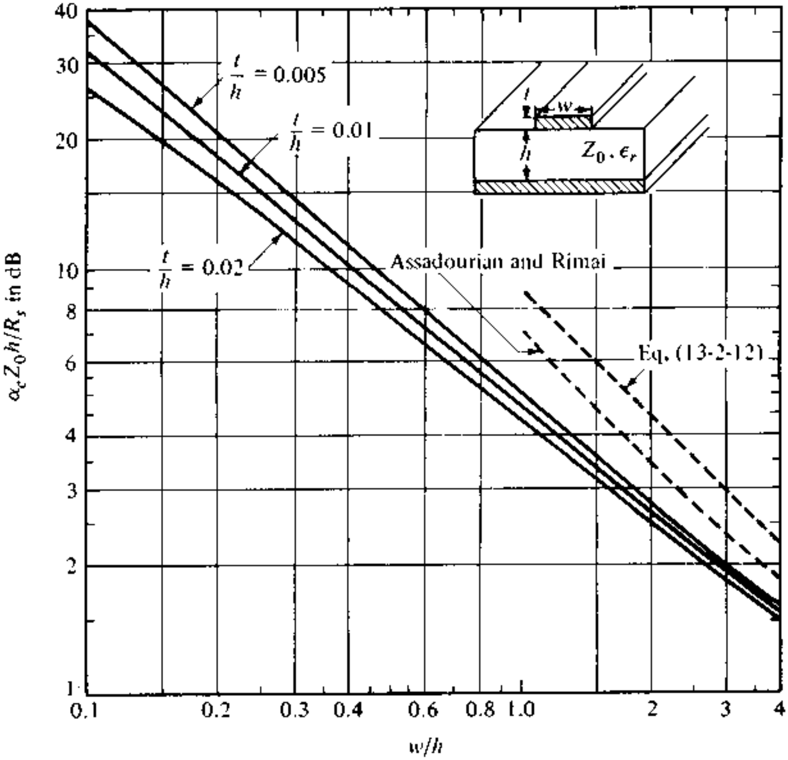

Figure 11-1-8 Theoretical conductor attenuation factor of microstrip as a function of w/h. (*After R. A. Pucel et al. [10]; reprinted by permission of IEEE, Inc.*)

Lewin's results show that the ratio of radiated power to total dissipated power for an open-circuited microstrip line is

$$\frac{P_{rad}}{P_t} = 240\pi^2 \left(\frac{h}{\lambda_0}\right)^2 \frac{F(\epsilon_{re})}{Z_0} \tag{11-1-28}$$

where $F(\epsilon_{re})$ is a radiation factor given by

$$F(\epsilon_{re}) = \frac{\epsilon_{re} + 1}{\epsilon_{re}} - \frac{\epsilon_{re} - 1}{2\epsilon_{re}\sqrt{\epsilon_{re}}} \ln \frac{\sqrt{\epsilon_{re}} + 1}{\sqrt{\epsilon_{re}} - 1} \tag{11-1-29}$$

in which ϵ_{re} is the effective dielectric constant and $\lambda_0 = c/f$ is the free-space wavelength.

The radiation factor decreases with increasing substrate dielectric constant. So, alternatively, Eq. (11-1-28) can be expressed as

$$\frac{P_{rad}}{P_t} = \frac{R_r}{Z_0} \tag{11-1-30}$$

where R_r is the radiation resistance of an open-circuited microstrip and is given by

$$R_r = 240\pi^2 \left(\frac{h}{\lambda_0}\right)^2 F(\epsilon_{re}) \tag{11-1-31}$$

The ratio of the radiation resistance R_r to the real part of the characteristic impedance Z_0 of the microstrip line is equal to a small fraction of the power radiated from a single open-circuit discontinuity. In view of Eq. (11-1-28), the radiation loss decreases when the characteristic impedance increases. For lower dielectric-constant substrates, radiation is significant at higher impedance levels. For higher dielectric-constant substrates, radiation becomes significant until very low impedance levels are reached.

11-1-3 Quality Factor Q of Microstrip Lines

Many microwave integrated circuits require very high quality resonant circuits. The quality factor Q of a microstrip line is very high, but it is limited by the radiation losses of the substrates and with low dielectric constant. Recall that for uniform current distribution in the microstrip line, the ohmic attenuation constant of a wide microstrip line is given by Eq. (11-1-21) as

$$\alpha_c = \frac{8.686R_s}{Z_0 w} \qquad \text{dB/cm}$$

and that the characteristic impedance of a wide microstrip line, as shown in Eq. (11-1-9), is

$$Z_0 = \frac{h}{w}\sqrt{\frac{\mu}{\epsilon}} = \frac{377}{\sqrt{\epsilon_r}}\frac{h}{w} \qquad \Omega$$

The wavelength in the microstrip line is

$$\lambda_g = \frac{30}{f\sqrt{\epsilon_r}} \qquad \text{cm} \tag{11-1-32}$$

where f is the frequency in GHz.

Since Q_c is related to the conductor attenuation constant by

$$Q_c = \frac{27.3}{\alpha_c} \tag{11-1-33}$$

where α_c is in dB/λ_g, Q_c of a wide microstrip line is expressed as

$$Q_c = 39.5\left(\frac{h}{R_s}\right) f_{\text{GHz}} \tag{11-1-34}$$

where h is measured in cm and R_s is expressed as

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = 2\pi\sqrt{\frac{f_{\text{GHz}}}{\sigma}} \qquad \Omega/\text{square}. \tag{11-1-35}$$

Finally, the quality factor Q_c of a wide microstrip line is

$$Q_c = 0.63h\sqrt{\sigma f_{\text{GHz}}} \tag{11-1-36}$$

where α is the conductivity of the dielectric substrate board in ℧/m.

For a copper strip, $\alpha = 5.8 \times 10^7$ ℧/m and Q_c becomes

$$Q_{\text{Cu}} = 4780h\sqrt{f_{\text{GHz}}} \tag{11-1-37}$$

For 25-mil alumina at 10 GHz, the maximum Q_c achievable from wide microstrip lines is 954 [13].

Similarly, a quality factor Q_d is related to the dielectric attenuation constant:

$$Q_d = \frac{27.3}{\alpha_d} \tag{11-1-38}$$

where α_d is in dB/λ_g.

Substituting Eq. (11-1-20) into Eq. (11-1-38) yields

$$Q_d = \frac{\lambda_0}{\sqrt{\epsilon_{re}}\tan\theta} \simeq \frac{1}{\tan\theta} \tag{11-1-39}$$

where λ_0 is the free-space wavelength in cm. Note that the Q_d for the dielectric attenuation constant of a microstrip line is approximately the reciprocal of the dielectric loss tangent θ and is relatively constant with frequency.

11-2 PARALLEL STRIP LINES

A parallel strip line consists of two perfectly parallel strips separated by a perfect dielectric slab of uniform thickness, as shown in Fig. 11-2-1. The plate width is w, the separation distance is d, and the relative dielectric constant of the slab is ϵ_{rd}.

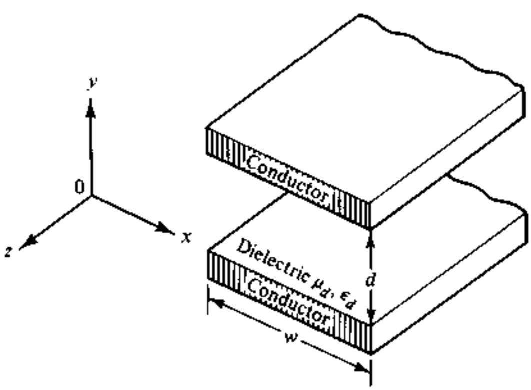

Figure 11-2-1 Schematic diagram of a parallel strip line.

11-2-1 Distributed Parameters

In a microwave integrated circuit a strip line can be easily fabricated on a dielectric substrate by using printed-circuit techniques. A parallel stripline is similar to a two-conductor transmission line, so it can support a quasi-TEM mode. Consider a TEM-mode wave propagating in the positive z direction in a lossless strip line ($R = G = 0$). The electric field is in the y direction, and the magnetic field is in the x direction. If the width w is much larger than the separation distance d, the fringing capacitance is negligible. Thus the equation for the inductance along the two conducting strips can be written as

$$L = \frac{\mu_c d}{w} \qquad \text{H/m} \tag{11-2-1}$$

where μ_c is the permeability of the conductor. The capacitance between the two conducting strips can be expressed as

$$C = \frac{\epsilon_d w}{d} \qquad \text{F/m} \tag{11-2-2}$$

where ϵ_d is the permittivity of the dielectric slab.

If the two parallel strips have some surface resistance and the dielectric substrate has some shunt conductance, however, the parallel stripline would have some losses. The series resistance for both strips is given by

$$R = \frac{2R_s}{w} = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} \qquad \Omega/\text{m} \tag{11-2-3}$$

where $R_s = \sqrt{(\pi f \mu_c)/\sigma_c}$ is the conductor surface resistance in Ω/square and σ_c is the conductor conductivity in ℧/m. The shunt conductance of the strip line is

$$G = \frac{\sigma_d w}{d} \qquad \text{℧/m} \tag{11-2-4}$$

where σ_d is the conductivity of the dielectric substrate.

11-2-2 Characteristic Impedance

The characteristic impedance of a lossless parallel strip line is

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{d}{w}\sqrt{\frac{\mu_d}{\epsilon_d}} = \frac{377}{\sqrt{\epsilon_{rd}}}\frac{d}{w} \qquad \text{for } w \gg d \tag{11-2-5}$$

The phase velocity along a parallel strip line is

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_d \epsilon_d}} = \frac{c}{\sqrt{\epsilon_{rd}}} \qquad \text{m/s} \qquad \text{for } \mu_c = \mu_0 \tag{11-2-6}$$

The characteristic impedance of a lossy parallel strip line at microwave frequencies ($R \ll \omega L$ and $G \ll \omega C$) can be approximated as

$$Z_0 \simeq \sqrt{\frac{L}{C}} = \frac{377}{\sqrt{\epsilon_{rd}}}\frac{d}{w} \qquad \text{for } w \gg d \tag{11-2-7}$$

11-2-3 Attenuation Losses

The propagation constant of a parallel strip line at microwave frequencies can be expressed by

$$\begin{aligned}\gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \qquad \text{for } R \ll \omega L \quad \text{and} \quad G \ll \omega C \\ &\simeq \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) + j\omega\sqrt{LC}\end{aligned} \tag{11-2-8}$$

Thus the attenuation and phase constants are

$$\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) \qquad \text{Np/m} \tag{11-2-9}$$

and

$$\beta = \omega\sqrt{LC} \qquad \text{rad/m} \tag{11-2-10}$$

Substitution of the distributed parameters of a parallel strip line into Eq.(11-2-9) yields the attenuation constants for the conductor and dielectric losses:

$$\alpha_c = \frac{1}{2}R\sqrt{\frac{C}{L}} = \frac{1}{d}\sqrt{\frac{\pi f \epsilon_d}{\sigma_c}} \qquad \text{Np/m} \tag{11-2-11}$$

and

$$\alpha_d = \frac{1}{2}G\sqrt{\frac{L}{C}} = \frac{188\sigma_d}{\sqrt{\epsilon_{rd}}} \qquad \text{Np/m} \tag{11-2-12}$$

Example 11-2-1: Characteristics of a Parallel Strip Line

A lossless parallel strip line has a conducting strip width w. The substrate dielectric separating the two conducting strips has a relative dielectric constant ϵ_{rd} of 6 (beryllia or beryllium oxide BeO) and a thickness d of 4 mm.

Calculate:

- **a.** The required width w of the conducting strip in order to have a characteristic impedance of 50 Ω
- **b.** The strip-line capacitance
- **c.** The strip-line inductance
- **d.** The phase velocity of the wave in the parallel strip line

Solution

a. From Eq. (11-2-5) the width of the conducting strip is

$$w = \frac{377}{\sqrt{\epsilon_{rd}}}\frac{d}{Z_0} = \frac{377}{\sqrt{6}}\frac{4 \times 10^{-3}}{50}$$

$$= 12.31 \times 10^{-3} \text{ m}$$

b. The strip-line capacitance is

$$C = \frac{\epsilon_d w}{d} = \frac{8.854 \times 10^{-12} \times 6 \times 12.31 \times 101^{-3}}{4 \times 10^{-3}}$$

$$= 163.50 \text{ pF/m}$$

c. The strip-line inductance is

$$L = \frac{\mu_c d}{w} = \frac{4\pi \times 10^{-7} \times 4 \times 10^{-3}}{12.31 \times 10^{-3}}$$

$$= 0.41 \ \mu\text{H/m}$$

d. The phase velocity is

$$v_p = \frac{c}{\sqrt{\epsilon_{rd}}} = \frac{3 \times 10^8}{\sqrt{6}}$$

$$= 1.22 \times 10^8 \text{ m/s}$$

11-3 COPLANAR STRIP LINES

A coplanar trip line consists of two conducting strips on one substrate surface with one strip grounded, as shown in Fig. 11-3-1. The coplanar strip line has advantages over the conventional parallel strip line (see Section 11-2) because its two strips are on the same substrate surface for convenient connections. In microwave integrated circuits (MICs) the wire bonds have always presented reliability and reproducibility problems. The coplanar strip lines eliminate the difficulties involved in connecting the shunt elements between the hot and ground strips. As a result, reliability is increased, reproducibility is enhanced, and production cost is decreased.

The characteristic impedance of a coplanar strip line is

$$Z_0 = \frac{2\, P_{avg}}{I_0^2} \tag{11-3-1}$$

where I_0 is the total peak current in one strip and P_{avg} is the average power flowing in the positive z direction. The average flowing power can be expressed as

$$P_{avg} = \tfrac{1}{2}\text{Re} \iint (\mathbf{E} \times \mathbf{H}^*) \cdot \mathbf{u}_z \, dx\, dy \tag{11-3-2}$$

where $\mathbf{E}_x$ = electric field intensity in the positive x direction
$\mathbf{H}_y$ = magnetic field intensity in the positive y direction
$*$ = conjugate

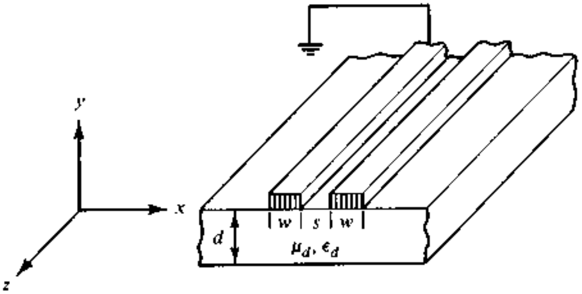

Figure 11-3-1 Schematic diagram of a coplanar strip line.

Example 11-3-1: Characteristic Impedance of a Coplanar Strip Line

A coplanar strip line carries an average power of 250 mW and a peak current of 100 mA. Determine the characteristic impedance of the coplanar strip line.

Solution From Eq. (11-3-1), the characteristic impedance of the coplanar strip line is

$$Z_0 = \frac{2 \times 250 \times 10^{-3}}{(100 \times 10^{-3})^2} = 50\ \Omega$$

11-4 SHIELDED STRIP LINES

A partially shielded strip line has its strip conductor embedded in a dielectric medium, and its top and bottom ground planes have no connection, as shown in Fig. 11-4-1.

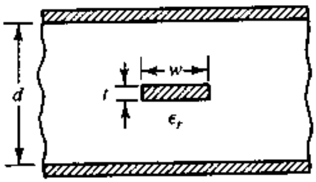

Figure 11-4-1 Partially shielded strip line.

The characteristic impedance for a wide strip ($w/d \gg 0.35$) [14] is

$$Z_0 = \frac{94.15}{\sqrt{\epsilon_r}}\left(\frac{w}{d}K + \frac{C_f}{8.854\epsilon_r}\right)^{-1} \quad (11\text{-}4\text{-}1)$$

where $K = \dfrac{1}{1 - t/d}$

t = the strip thickness

d = the distance between the two ground planes

$C_f = \dfrac{8.854\epsilon_r}{\pi}[2K \ln (K + 1) - (K - 1) \ln (K^2 - 1)]$ and is the fringe capacitance in pF/m

Figure 11-4-2 shows the characteristic impedance $\mathbf{Z}_0$ for a partially shielded strip line, with the t/d radio as a parameter.

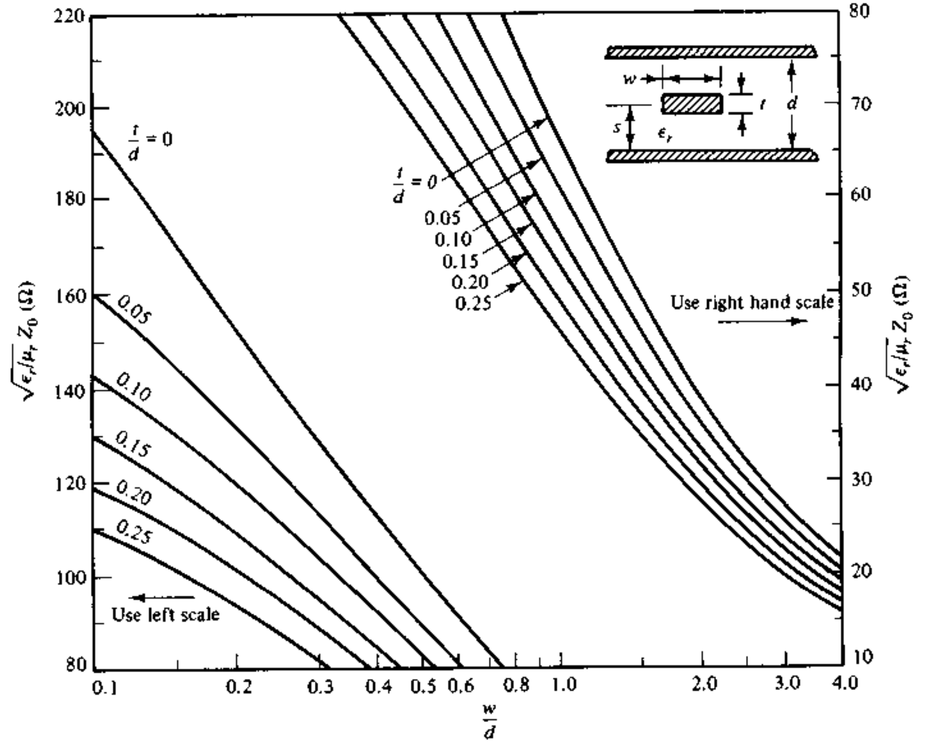

Figure 11-4-2 Characteristic impedance $\mathbf{Z}_0$ of a partially shielded strip line with the t/d ratio as a parameter. (*After S. Cohn [14]; reprinted by permission of IEEE, Inc.*)

Example 11-4-1: Characteristic Impedance of a Shielded Strip Line

A shielded strip line has the following parameters:

Dielectric constant of the insulator (polystyrene):	$\epsilon_r = 2.56$
Strip width:	$w = 25$ mils
Strip thickness:	$t = 14$ mils
Shield depth:	$d = 70$ mils

Calculate:

a. The K factor

b. The fringe capacitance

c. The characteristic impedance of the line.

Solution

a. Using Eq. (11-4-1), the K factor is obtained:

$$K = \left(1 - \frac{t}{d}\right)^{-1} = \left(\frac{1 - 14}{70}\right)^{-1} = 1.25$$

b. From Eq. (11-4-1), the fringe capacitance is

$$C_f = \frac{8.854 \times 2.56}{3.1416}[2 \times 1.25 \ln (1.25 + 1) - (1.25 - 1) \ln (1.25^2 - 1)]$$
$$= 15.61 \text{ pF/m}.$$

c. The characteristic impedance from Eq. (11-4-1) is

$$Z_0 = \frac{94.15}{\sqrt{2.56}}\left[\frac{25}{70}(1.25) + \frac{15.61}{8.854 \times 2.56}\right]^{-1}$$
$$= 50.29\ \Omega$$

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PROBLEMS

Microstrip Lines

11-1. A microstrip line has the following parameters:

ϵ_r = 5.23 and is the relative dielectric constant of the fiberglass board material
h = 0.8 mils
t = 2.8 mils
w = 10 mils

Write a FORTRAN program to complete the characteristic impedance Z_0 of the line. Use a READ statement to read in the input values, the F10.5 format for numerical outputs, and the Hollerith format for character outputs.

11-2. Since modes on microstrip lines are only quasi-transverse electric and magnetic (TEM), the theory of TEM-coupled lines applies only approximately. From the basic theory of a lossless line, show that the inductance L and capacitance C of a microstrip line are

$$L = \frac{Z_0}{v} = \frac{Z_0\sqrt{\epsilon_r}}{c}$$

and

$$C = \frac{1}{Z_0 v} = \frac{\sqrt{\epsilon_r}}{Z_0 c}$$

where Z_0 = characteristic impedance of the microstrip line
v = wave velocity in the microstrip line
$c = 3 \times 10^8$ m/s, the velocity of light in vacuum
ϵ_r = relative dielectric constant of the board material

11-3. A microstrip line is constructed of a perfect conductor and a lossless dielectric board. The relative dielectric constant of the fiberglass-epoxy board is 5.23, and the line characteristic impedance is 50 Ω. Calculate the line inductance and the line capacitance.

11-4. A microstrip line is constructed of a copper conductor and nylon phenolic board. The relative dielectric constant of the board material is 4.19, measured at 25 GHz, and its thickness is 0.4836 mm (19 mils). The line width is 0.635 mm (25 mils), and the line thickness is 0.071 mm (2.8 mils). Calculate the

a. Characteristic impedance Z_0 of the microstrip line
b. Dielectric filling factor q
c. Dielectric attenuation constant α_d
d. Surface skin resistivity R_s of the copper conductor at 25 GHz
e. Conductor attenuation constant α_c

11-5. A microstrip line is made of a copper conductor 0.254 mm (10 mils) wide on a G-10 fiberglass-epoxy board 0.20 mm (8 mils) in height. The relative dielectric constant ϵ_r of the board material is 4.8, measured at 25 GHz. The microstrip line 0.035-mm (1.4 mils) thick is to be used for 10 GHz. Determine the
a. Characteristic impedance Z_0 of the microstrip line
b. Surface resistivity R_s of the copper conductor
c. Conductor attenuation constant α_c
d. Dielectric attenuation constant α_d
e. Quality factors Q_c and Q_d

Parallel Striplines

11-6. A gold parallel stripline has the following parameters:

Relative dielectric constant of teflon:	$\epsilon_{rd} = 2.1$
Strip width:	$w = 26$ mm
Separation distance:	$d = 5$ mm
Conductivity of gold:	$\sigma_c = 4.1 \times 10^7$ ℧/m
Frequency:	$f = 10$ GHz

Determine the
a. Surface resistance of the gold strip
b. Characteristic impedance of the strip line
c. Phase velocity

11-7. A gold parallel strip line has the following parameters:

Relative dielectric constant of polyethylene:	$\epsilon_{rd} = 2.25$
Strip width:	$w = 25$ mm
Separation distance:	$d = 5$ mm

Calculate the
a. Characteristic impedance of the strip line
b. Strip-line capacitance
c. Strip-line inductance
d. Phase velocity

Coplanar Strip Lines

11-8. A 50-Ω coplanar strip line has the following parameters:

Relative dielectric constant of alumina:	$\epsilon_{rd} = 10$
Strip width:	$w = 4$ mm
Strip thickness:	$t = 1$ mm

TEM-mode field intensities: $E_y = 3.16 \times 10^3 \sin\left(\frac{\pi x}{w}\right) e^{-j\beta z}$

$$H_x = 63.20 \sin\left(\frac{\pi x}{w}\right) e^{-j\beta z}$$

Find the

a. Average power flow

b. Peak current in one strip

11-9. A shielded stripline has the following parameters:

Relative dielectric constant of the insulator polyethylene:	$\epsilon_{rd} = 2.25$
Strip width:	$w = 2$ mm
Strip thickness:	$t = 0.5$ mm
Shield depth:	$d = 4$ mm

Calculate the

a. K factor

b. Fringe capacitance

c. Characteristic impedance

11-10. A shielded strip line is made of a gold strip in a polystyrene dielectric insulator and has the following parameters:

Relative dielectric constant of polystyrene:	$\epsilon_{rd} = 2.56$
Strip width:	$w = 0.7$ mm
Strip thickness:	$t = 1.4$ mm
Shield depth:	$d = 3.5$ mm

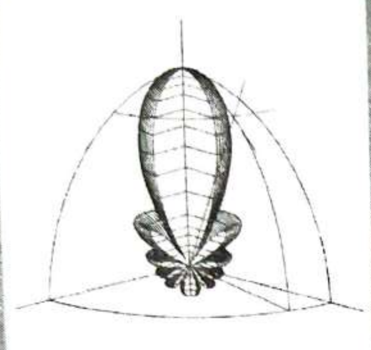
Determine the

a. K factor

b. Fringe capacitance

c. Characteristic impedance

Chapter 2



Antenna Basics

This chapter includes the following topics:

- Basic parameters
- Patterns
- Beam area
- Radiation intensity
- Beam efficiency
- Directivity and gain
- Directivity and resolution
- Antenna apertures and radar cross-section
- Effective height
- Radio links (Friis formula)
- Field from oscillating dipole
- Antenna field zones
- Shape-impedance consideration
- Polarization
- Polarization ellipse and Poincare sphere
- Signal-to-noise ratio
- Antenna temperature
- Antenna impedance

2–1 Introduction

The world of antennas encompasses the understanding of the meaning, purpose, parameters and types of antennas. It also spells the theoretical and practical aspects of antennas and their selection criterion for specific applications and range of operations.

An antenna (aerial) is considered as a region of transition between a transmission line and space. Antennas radiate/couple/concentrate/direct electromagnetic energy in the desired/assigned direction. An antenna may be *isotropic* (also called omni-directional/omni/non-directional) or *anisotropic* (directional).

There is no hard and fast rule for selecting an antenna for a particular frequency range or application. While choosing an antenna, many electrical, mechanical and structural aspects are to be taken into account. These aspects include radiation pattern, gain, efficiency, impedance, frequency characteristics, shape, size, weight and look of antennas, and above all their economic viability.

In some applications (e.g., radars, mobiles), the same antenna may be used for transmission and reception, while in others (e.g., radio and television) transmission and reception of signals require separate antennas which differ in shape and size and other characteristics. In principle, there is no difference in selection factors relating to transmitting and receiving antennas. The cost, shape and size, etc., make the main difference. Still, high efficiency and high gain are the basic requirements for transmitting antennas, whereas low side lobes and large signal-to-noise ratio are the key selection criteria for receiving antennas.

Antennas may vary in size from the order of a few millimeters (strip antennas in cellular phones) to 1000's of feet (dish antennas for astronomical observations). Besides, 1D linear arrays and 2D planar arrays with lengths or diameters of tens of kilometers operate at the lower edge of the frequency spectrum.

This chapter shall make you well-versed in the language of antennas and comfortable with their culture.

2–2 Basic Antenna Parameters

A *radio antenna* may be defined as the structure associated with the region of transition between a guided wave and a free-space wave, or vice versa. Antennas convert electrons to photons, or vice versa.[1]

Regardless of antenna type, all involve the same basic principle that radiation is produced by accelerated (or decelerated) charge. The *basic equation of radiation* may be expressed simply as

$$\dot{I}L = Q\dot{v} \qquad (\mathrm{A\,m\,s^{-1}}) \text{ Basic radiation equation} \tag{1}$$

where

$\dot{I}$ = time-changing current, A s^{-1}
L = length of current element, m
Q = charge, C
$\dot{v}$ = time change of velocity which equals the acceleration of the charge, m s^{-2}

Thus, ***time-changing current and accelerated charge radiates.*** For steady-state harmonic variation, we usually focus on current. For transients or pulses, we focus on charge.[2] The radiation is perpendicular to the acceleration, and the radiated power is proportional to the square of $\dot{I}L$ or $Q\dot{v}$.

The two-wire transmission line in Fig. 2–1*a* is connected to a radio-frequency generator (or transmitter). Along the uniform part of the line, energy is guided as a plane Transverse ElectroMagnetic Mode (TEM) wave with little loss. The spacing between wires is assumed to be a small fraction of a wavelength. Further on, the transmission line opens out in a tapered transition. As the separation approaches the order of a wavelength or more, the wave tends to be radiated so that the opened-out line acts like an antenna which launches a free-space wave. The currents on the transmission line flow out on the antenna and end there, but the fields associated with them keep on going.

The study of transmission lines reveals that there would be perfect reflection of a wave if it is open circuited (OC) or short circuited (SC). In fact, reflections are present even if there is a slight mismatch on account of termination or imperfections in the transmission path itself. An equivalent circuit of a line with loss can be drawn in terms of its resistance (R), inductance (L) and capacitance (C) or only in terms of L and C for a lossless line. The burden of carrying energy (contents) of a propagating wave is shared by electric (E) and magnetic (H) fields or voltage (v) and current (i) alike. The portions of energy shared by each are accounted by the well-known relations $Cv^2/2$ (for electric field) and $Li^2/2$ (for magnetic filed).

Consider a wave propagating down a line finding an open-circuited end. As the wave arrives at the OC end, the current becomes zero and part of the energy shared by magnetic field becomes (mathematically) zero. But since the energy can neither be created nor destroyed, the only way out is that this burden is also taken up by the surviving (electric) field. In the expression $Cv^2/2$, the line parameter C ($= \varepsilon A/d$) cannot change unless the area of cross-section 'A', the separation 'd' or the permittivity of the material occupying the space in the line configuration gets altered. The change of voltage is the only possibility by which the additional energy can be carried by the electric field. Thus, the voltage rises at the OC end to enable it to carry the total energy. The voltage at a point of the OC is now higher than just before a little distance towards the sending end. The

[1]A photon is a quantum unit of electromagnetic energy equal to hf, where h = Planck's constant ($= 6.63 \times 10^{-34}$ J s) and f = frequency (Hz).

[2]A pulse radiates with a broad bandwidth (the shorter the pulse the broader the bandwidth). A sinusoidal variation results in a narrow bandwidth (theoretically zero at the frequency of the sinusoid if it continues indefinitely).

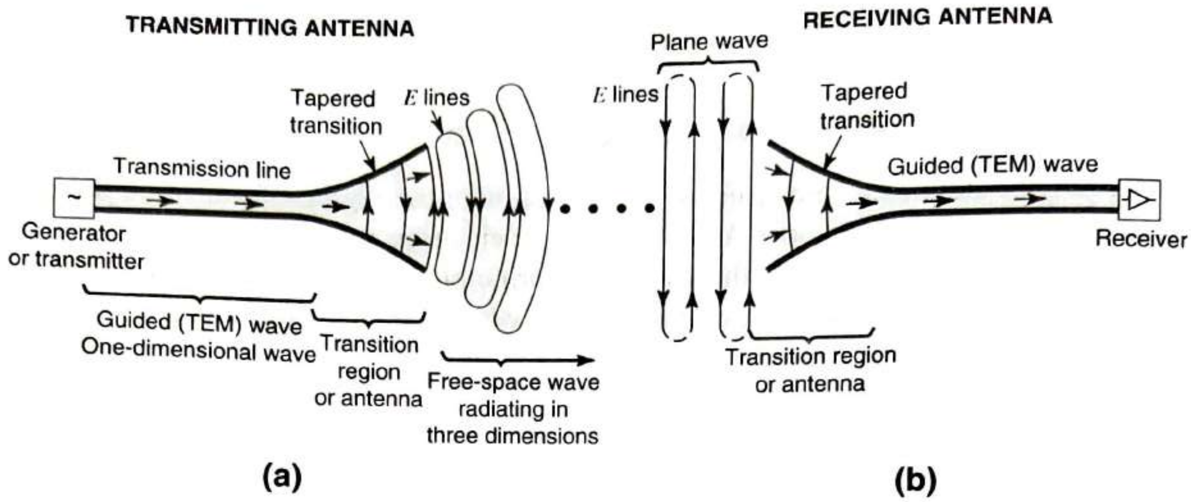


Figure 2–1 (*a*) Radio (or wireless) communication link with transmitting antenna and (*b*) receiving antenna. The receiving antenna is remote from the transmitting antenna so that the spherical wave radiated by the transmitting antenna arrives as an essentially plane wave at the receiving antenna.

current starts flowing back, which amounts to traveling of the current wave towards the sending end. The voltage wave joins the race as it was earlier, but this time both (current and voltage) waves are the reflected ones and share their own burdens as before. In fact, the current momentarily remains zero and that too only at the OC point. Similarly, if the line is short-circuited at the receiving end, the voltage (and hence electric field) becomes zero and part of energy shared by it is momentarily shared by the magnetic field. This time the current rises at the receiving end resulting in initiation of a voltage wave followed by a current wave as discussed above.

In cases of perfect open circuits or perfect short circuits, theoretically there must be perfect reflection. Since the wave possesses a moment-of-inertia-like property, it will take some time to change its direction. However small this time may be, some energy is likely to leak in to the space. This process of leakage can be termed as ***radiation***. This phenomenon is better understood through the example of an open-circuited parallel wire line. The more is the opening at the end of the line, the more time will be taken by the wave to change its direction and thus more energy will leak in to the space or there will be more coupling of transmission line to the space. The maximum radiation will, therefore, occur when the two wires at the end are flared to form a 180° angle. It is this process which has been illustrated in Fig. 2–1*a*.

The transmitting antenna in Fig. 2–1*a* is a region of transition from a guided wave on a transmission line to a free-space wave. The receiving antenna (Fig. 2–1*b*) is a region of transition from a space wave to a guided wave on a transmission line. Thus, ***an antenna is a transition device, or transducer, between a guided wave and a free-space wave, or vice-versa.*** The antenna is a device which interfaces a circuit and space.

From the circuit point of view, the antennas appear to the transmission lines as a resistance R_r, called the *radiation resistance*. It is not related to any resistance in the antenna itself but is a resistance coupled from space to the antenna terminals.

In the transmitting case, the radiated power is absorbed by objects at a distance: trees, buildings, the ground, the sky, and other antennas. In the receiving case, passive radiation from distant objects or active radiation from other antennas raises the apparent temperature of R_r. For lossless antennas this temperature has nothing to do with the physical temperature of the antenna itself but is related to the temperature of distant objects that the antenna is "looking at," as suggested in Fig. 2–2. In this sense, a receiving antenna (and its associated receiver) may be regarded as a remote-sensing temperature-measuring device.

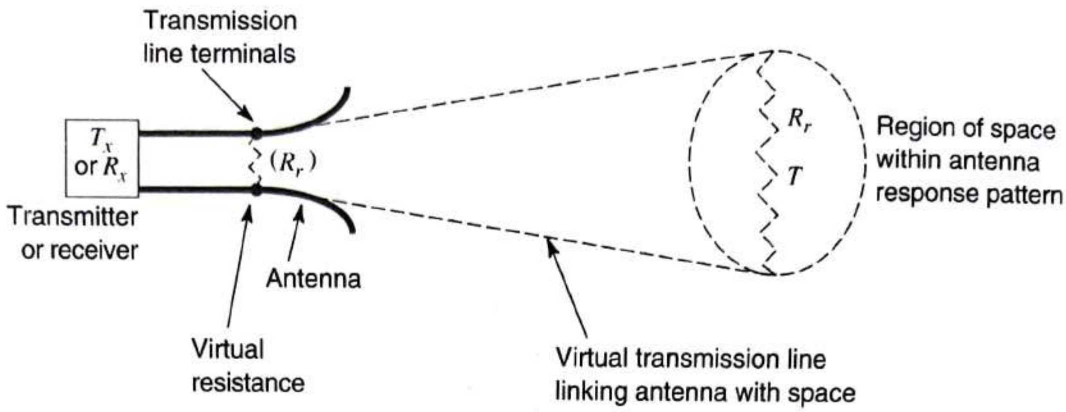


Figure 2–2 Schematic representation of region of space at temperature T linked via a virtual transmission line to an antenna.

As pictured schematically in Fig. 2–2, the radiation resistance R_r may be thought of as a "virtual" resistance that does not exist physically but is a quantity coupling the antenna to distant regions of space via a "virtual" transmission line.[1]

2–3 Patterns

Both the radiation resistance R_r, and its temperature T_A are simple scalar quantities. The radiation patterns, on the other hand, are three-dimensional quantities involving the variation of field or power (proportional to the field squared) as a function of the spherical coordinates θ and ϕ. Figure 2–3 shows a three-dimensional field pattern with pattern radius r (from origin to pattern boundary at the dot) proportional to the field intensity in the direction θ and ϕ. The pattern has its *main lobe* (maximum radiation) in the z direction ($\theta = 0$) with *minor lobes* (side and back) in other directions.

To completely specify the radiation pattern with respect to field intensity and polarization requires three patterns:

1. The θ component of the electric field as a function of the angles θ and ϕ or $E_\theta(\theta, \phi)$ (V m^{-1}) as in Figs. 2–3 and 2–4.
2. The ϕ component of the electric field as a function of the angles θ and ϕ or $E_\phi(\theta, \phi)$ (V m^{-1}).
3. The phases of these fields as a function of the angles θ and ϕ or $\delta_\theta(\theta, \phi)$ and $\delta_\phi(\theta, \phi)$ (rad or deg).

Any field pattern can be presented in three-dimensional spherical coordinates, as in Fig. 2–3, or by plane cuts through the main-lobe axis. Two such cuts at right angles, called the *principal plane patterns* (as in the xz and yz planes in Fig. 2–3) may be required but if the pattern is symmetrical around the z axis, one cut is sufficient.

Figures 2–4*a* and 2–4*b* are principal plane field and power patterns in polar coordinates. The same pattern is presented in Fig. 2–4*c* in rectangular coordinates on a logarithmic, or decibel, scale which gives the minor lobe levels in more detail.

[1] It is to be noted that the radiation resistance, the antenna temperature, and the radiation patterns are functions of the frequency. In general, the patterns are also functions of the distance at which they are measured, but at distances which are large compared to the size of the antenna and large compared to the wavelength, the pattern is independent of distance. Usually the patterns of interest are for this far-field condition.

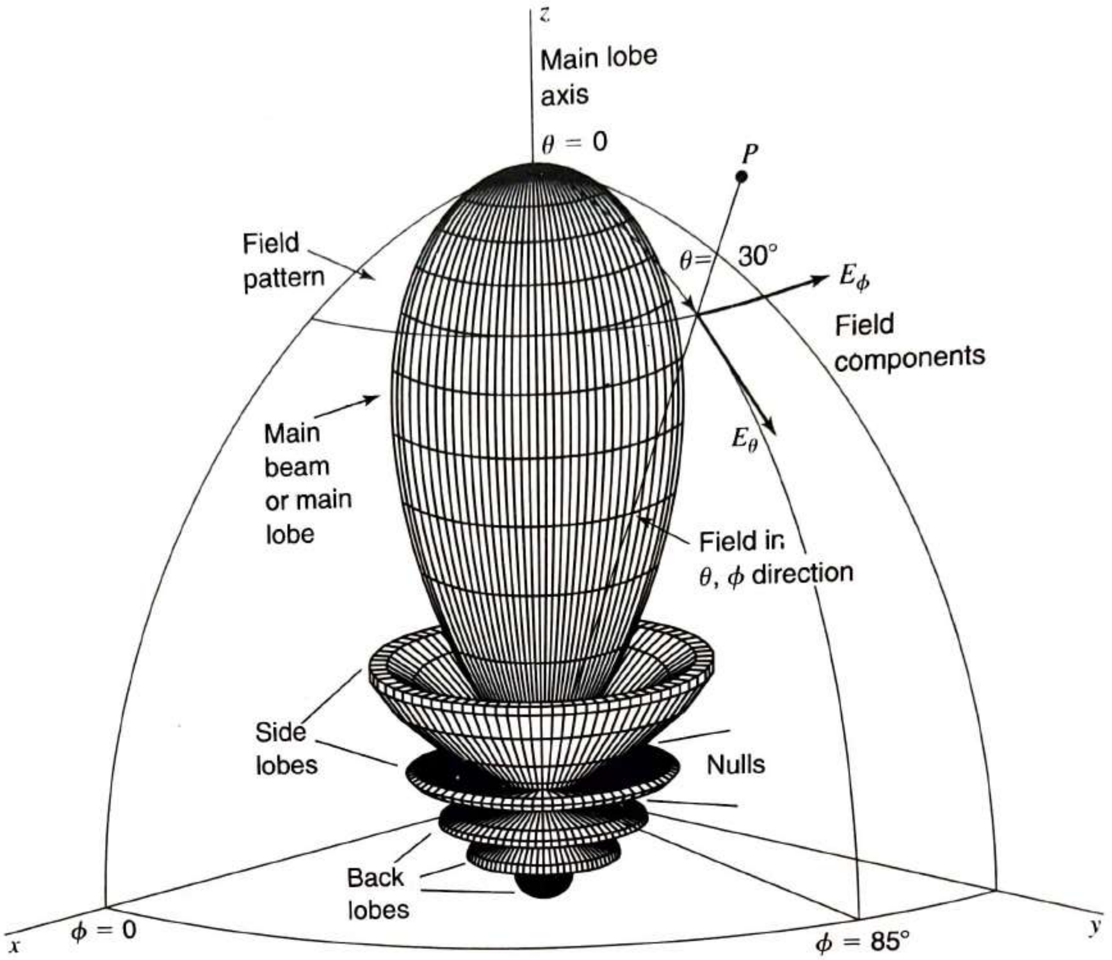


Figure 2–3 Three-dimensional field pattern of a directional antenna with maximum radiation in z-direction at $\theta = 0°$. Most of the radiation is contained in a *main beam* (or *lobe*) accompanied by radiation also in *minor lobes* (*side* and *back*). Between the lobes are *nulls* where the field goes to zero. The radiation in any direction is specified by the angles θ and ϕ. The direction of the point P is at the angles $\theta = 30°$ and $\phi = 85°$. This pattern is symmetrical in ϕ and a function only of θ.

The angular beamwidth at the half-power level or *half-power beamwidth* (HPBW) (or −3-dB beamwidth) and the *beamwidth between first nulls* (FNBW) as shown in Fig. 2–4, are important pattern parameters.

Dividing a field component by its maximum value, we obtain a *normalized or relative field pattern* which is a dimensionless number with maximum value of unity. Thus, the normalized field pattern (Fig. 2–4*a*) for the electric field is given by

$$\textbf{\textit{Normalized field pattern}} = E_\theta(\theta,\phi)_n = \frac{E_\theta(\theta,\phi)}{E_\theta(\theta,\phi)_{\max}} \qquad \text{(dimensionless)} \tag{1}$$

The half-power level occurs at those angles θ and ϕ for which $E_\theta(\theta,\phi)_n = 1/\sqrt{2} = 0.707$.

At distances that are large compared to the size of the antenna and large compared to the wavelength, the shape of the field pattern is independent of distance. Usually the patterns of interest are for this *far-field* condition.

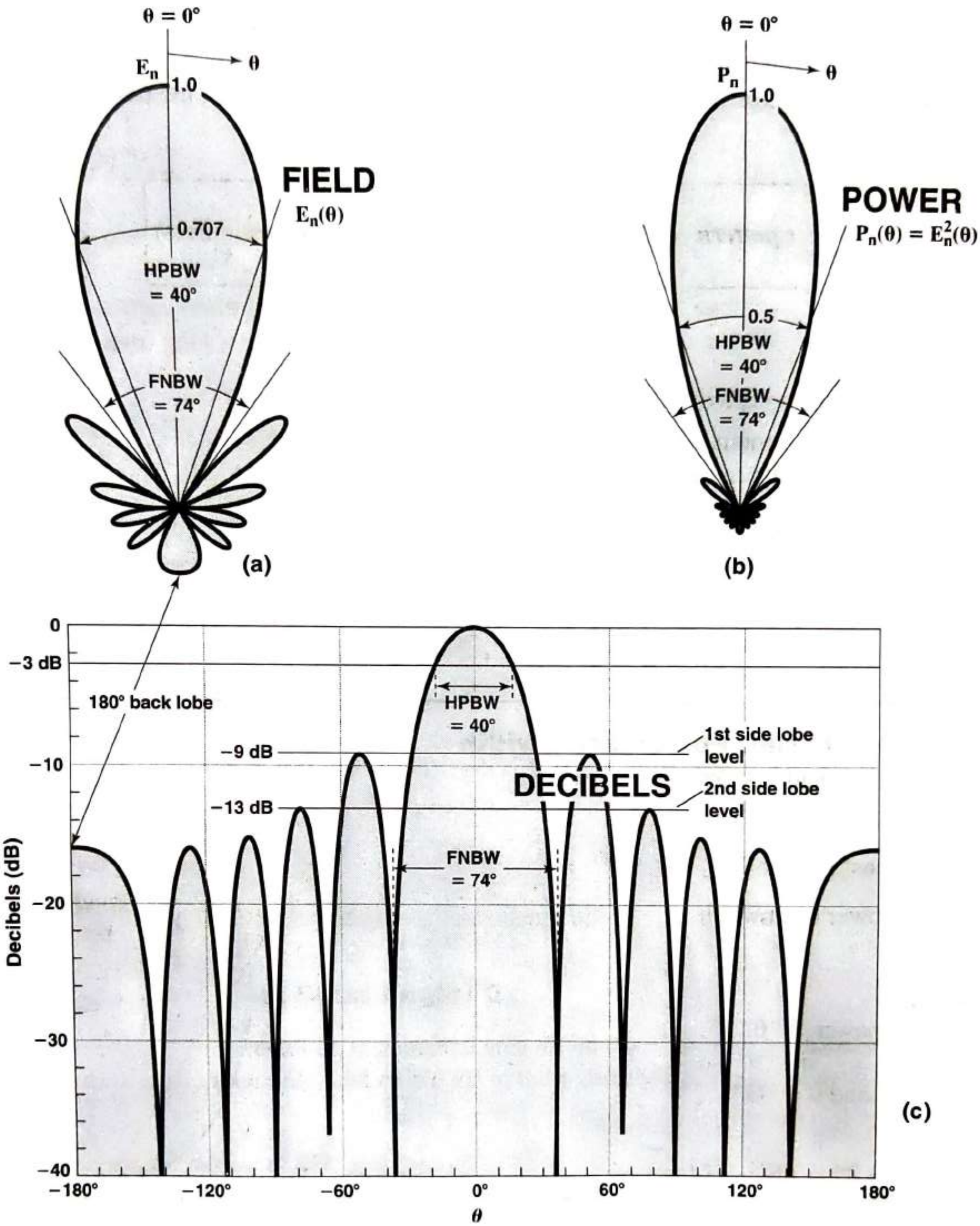


Figure 2–4 Two-dimensional field power and decibel plots of the 3-D antenna pattern of Fig. 2–3. Taking a slice through the middle of the 3-dimensional pattern of Fig. 2–3 results in the 2-dimensional pattern at (*a*). It is a field pattern (proportional to the electric field E in V/m) with normalized relative field $E_n(\theta) = 1$ at $\theta = 0°$. The half-power beam width (HPBW) $= 40°$ is measured at the $E = 0.707$ level. The pattern at (*b*) is a power plot of (*a*) (proportional to E^2) with relative power $P_n = 1$ at $\theta = 0°$ and with HPBW $= 40°$ as before and measured at the $P_n = 0.5$ level. A decibel (dB) plot of (*a*) is shown at (*c*) with HPBW $= 40°$ as before and measured at the -3 dB level. The first side lobes are shown at the -9 dB and second side lobes at -13 dB. Decibel plots are useful for showing minor lobe levels.

Patterns may also be expressed in terms of the *power per unit area* [or Poynting vector $S(\theta, \phi)$].[1] Normalizing this power with respect to its maximum value yields a *normalized power pattern* as a function of angle which is a dimensionless number with a maximum value of unity. Thus, the normalized power pattern (Fig. 2–4*b*) is given by

$$\textit{Normalized power pattern} = P_n(\theta, \phi)_n = \frac{S(\theta, \phi)}{S(\theta, \phi)_{\max}} \quad \text{(dimensionless)} \tag{2}$$

where

$S(\theta, \phi)$ = Poynting vector = $\left[E_\theta^2(\theta, \phi) + E_\phi^2(\theta, \phi)\right]/Z_0$, W m^{-2}
$S(\theta, \phi)_{\max}$ = maximum value of $S(\theta, \phi)$, W m^{-2}
Z_0 intrinsic impedance of space =376.7 Ω

The decibel level is given by

$$\text{dB} = 10 \log_{10} P_n(\theta, \phi) \tag{3}$$

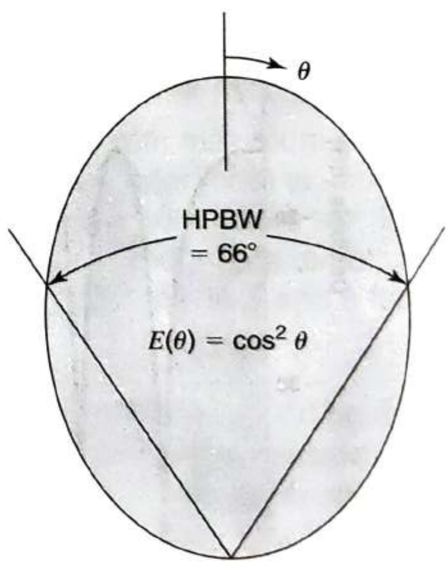
where $P_n(\theta, \phi)$ is as given by (2).

EXAMPLE 2–3.1 Half-Power Beamwidth

An antenna has a field pattern given by

$$E(\theta) = \cos^2\theta \quad \text{for } 0^\circ \le \theta \le 90^\circ$$

Find the half-power beamwidth (HPBW).



■ Solution

$E(\theta)$ at half power = 0.707. Thus $0.707 = \cos^2\theta$ so $\cos\theta = \sqrt{0.707}$ and $\theta = 33^\circ$

$$\text{HPBW} = 2\theta = 66^\circ \qquad \textit{Ans.}$$

EXAMPLE 2–3.2 Half-Power Beamwidth and First Null Beamwidth

An antenna has a field pattern given by $E(\theta) = \cos\theta\cos 2\theta$ for $0^\circ \le \theta \le 90^\circ$. Find (*a*) the half-power beamwidth (HPBW) and (*b*) the beamwidth between first nulls (FNBW).

[1]Although the Poynting vector, as the name implies, is a vector (with magnitude and direction), we use here its magnitude; its direction is the far field is radially outward.

■ **Solution**

(*a*) $E(\theta)$ at half power $= 0.707$. Thus $0.707 = \cos\theta\cos 2\theta = 1/\sqrt{2}$.

$$\cos 2\theta = \frac{1}{\sqrt{2}\cos\theta} \qquad 2\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta}\right) \qquad \text{and}$$

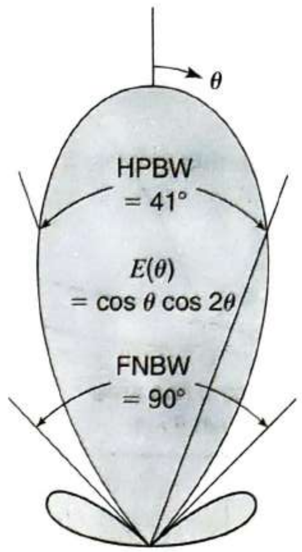
$$\theta = \frac{1}{2}\cos^{-1}\left(\frac{1}{\sqrt{2}\cos\theta'}\right)$$

Iterating with $\theta' = 0$ as a first guess, $\theta = 22.5^\circ$. Setting $\theta' = 22.5^\circ$, $\theta = 20.03^\circ$, etc., until after next iteration $\theta = \theta' = 20.47^\circ \cong 20.5^\circ$ and

$$\text{HPBW} = 2\theta = 41^\circ \qquad \textit{Ans. (a)}$$

(*b*) $0 = \cos\theta\cos 2\theta$, so $\theta = 45^\circ$ and

$$\text{FNBW} = 2\theta = 90^\circ \qquad \textit{Ans. (b)}$$



Although the radiation pattern characteristics of an antenna involve three-dimensional vector fields for a full representation, several simple single-valued scalar quantities can provide the information required for many engineering applications. These are:

- Half-power beamwidth, HPBW
- Beam area, Ω_A
- Beam efficiency, ε_M
- Directivity D or gain G
- Effective aperture A_e

The half-power beamwidth was discussed above. The others follow.

2–4 Beam Area (or Beam Solid Angle) Ω_A

In polar two-dimensional coordinates an incremental area dA on the surface of a sphere is the product of the length $r\,d\theta$ in the θ direction (latitude) and $r\sin\theta\,d\phi$ in the ϕ direction (longitude), as shown in Fig. 2–5.

Thus,

$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2\,d\Omega \tag{1}$$

where

$d\Omega =$ *solid angle* expressed in steradians (sr) or square degrees($\square$)

$d\Omega =$ solid angle subtended by the area dA

The area of the strip of width $r\,d\theta$ extending around the sphere at a constant angle θ is given by $(2\pi r\sin\theta)(r\,d\theta)$. Integrating this for θ values from 0 to π yields the area of the sphere. Thus,

$$\text{Area of sphere} = 2\pi r^2\int_0^\pi \sin\theta\,d\theta = 2\pi r^2[-\cos\theta]_0^\pi = 4\pi r^2 \tag{2}$$

where $4\pi =$ solid angle subtended by a sphere, sr

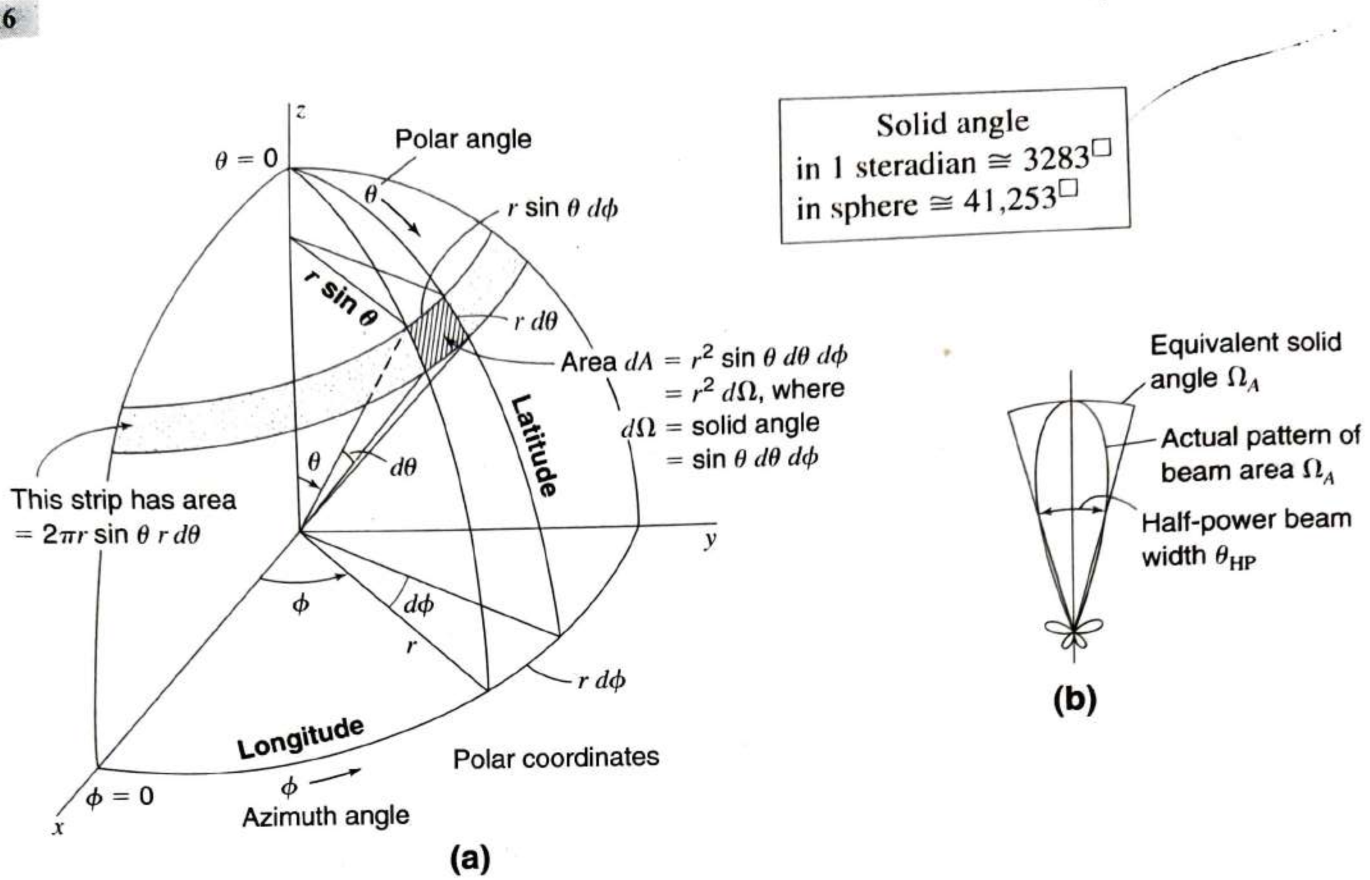


Figure 2–5 Polar coordinates showing incremental solid angle $dA = r^2\,d\Omega$ on the surface of a sphere of radius r where $d\Omega$ = solid angle subtended by the area dA. (*b*) Antenna power pattern and its equivalent solid angle or beam area Ω_A.

Thus,

$$1 \text{ steradian} = 1 \text{ sr} = (\text{solid angle of sphere})/(4\pi)$$

$$= 1 \text{ rad}^2 = \left(\frac{180}{\pi}\right)^2 (\text{deg}^2) = 3282.8064 \text{ square degrees} \tag{3}$$

Therefore,

$$4\pi \text{ steradians} = 3282.8064 \times 4\pi = 41{,}252.96 \cong 41{,}253 \text{ square degrees} = 41{,}253^{\square}$$

$$= \text{solid angle in a sphere} \tag{4}$$

The beam area or *beam solid angle* or Ω_A of an antenna (Fig. 2–5*b*) is given by the integral of the normalized power pattern over a sphere (4π sr)

$$\Omega_A = \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} P_n(\theta, \phi) \sin\theta \, d\theta \, d\phi \tag{5a}$$

and

$$\boxed{\Omega_A = \iint_{4\pi} P_n(\theta, \phi)\, d\Omega \qquad (\text{sr}) \qquad \textbf{\textit{Beam area}}} \tag{5b}$$

where $d\Omega = \sin\theta \, d\theta \, d\phi$, sr.

The beam area Ω_A is the solid angle through which all of the power radiated by the antenna would stream if $P(\theta, \phi)$ maintained its maximum value over Ω_A and was zero elsewhere. Thus the power radiated $= P(\theta, \phi)\Omega_A$ watts.

The *beam area* of an antenna can often be described *approximately* in terms of the angles subtended by the *half-power points* of the main lobe in the two principal planes. Thus,

$$\textbf{Beam area} \cong \Omega_A \cong \theta_{HP}\phi_{HP} \quad \text{(sr)} \tag{6}$$

where θ_{HP} and ϕ_{HP} are the *half-power beamwidths* (HPBW) in the two principal planes, minor lobes being neglected.

EXAMPLE 2-4.1 Solid Angle of Area in Square Degrees

Find the number of square degrees in the solid angle Ω on a spherical surface that is between $\theta = 20°$ and $\theta = 40°$ (or 70° and 50° north latitude) and between $\phi = 30°$ and $\phi = 70°$ (30° and 70° east longitude).

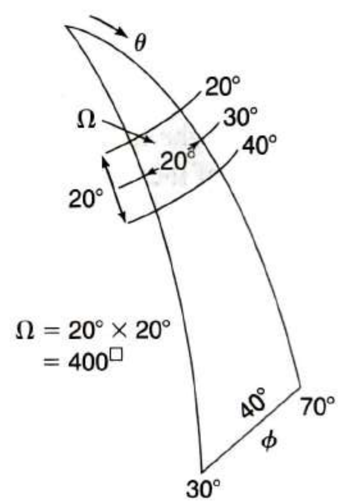
■ Solution

From (1)

$$\Omega = \int_{30°}^{70°} d\phi \int_{20°}^{40°} \sin\theta \, d\theta = \frac{40}{360} 2\pi[-\cos\theta]_{20}^{40}$$

$$= 0.222\pi \times 0.173 = 0.121 \text{ steradians} \quad \text{(sr)}$$

$$= 0.121 \times 3283 = 397 \text{ square degrees} = 397^{\square} \quad \textit{Ans.}$$



The solid angle Ω shown in the sketch may be *approximated* as the product of two angles $\Delta\theta = 20°$ and $\Delta\phi = 40° \sin 30° = 40° \times 0.5 = 20°$ where 30° is the median θ value of latitude. Thus, $\Omega = \Delta\theta\,\Delta\phi = 20° \times 20° = 400^{\square}$, which is within 3/4% of the answer given above.

EXAMPLE 2-4.2 Beam Area Ω_A of Antenna with $\cos^2\theta$ Pattern

An antenna has a field pattern given by $E(\theta) = \cos^2\theta$ for $0° \le \theta \le 90°$. This is the same pattern of Example 2-3.1. Find the beam area of this pattern.

■ Solution

From (5)

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} \cos^4\theta \sin\theta \, d\theta \, d\phi$$

$$= -2\pi\left[\frac{1}{25}\cos^5\theta\right]_0^{\pi/2} = \frac{2\pi}{5} = 1.26 \text{ sr} \quad \textit{Ans.}$$

From (6) an *approximate* relation for the beam area

$$\Omega_A \cong \theta_{HP}\phi_{HP} \quad \text{(sr)}$$

where θ_{HP} and ϕ_{HP} are the *half-power beamwidths* (HPBW) in the two principal planes. From Example 2–3.1, $\theta_{HP} = \phi_{HP} = 66°$, so

$$\Omega_A \cong \theta_{HP}\phi_{HP} = 66^2 = 4356 \text{ sq deg} = 4356^{\square}$$

From (3), one square radian = 3283 sq deg so

$$\text{Beam area } \Omega_A = 4356/3282 = 1.33 \text{ sr} \qquad \textit{Approx. Ans.}$$

a difference of 6%.

2–5 Radiation Intensity

The power radiated from an antenna per unit solid angle is called the *radiation intensity* U (watts per steradian or per square degree). The normalized power pattern of the previous section can also be expressed in terms of this parameter as the ratio of the radiation intensity $U(\theta, \phi)$, as a function of angle, to its maximum value. Thus,

$$P_n(\theta, \phi) = \frac{U(\theta, \phi)}{U(\theta, \phi)_{\max}} = \frac{S(\theta, \phi)}{S(\theta, \phi)_{\max}} \tag{1}$$

Whereas the Poynting vector S depends on the distance from the antenna (varying inversely as the square of the distance), the radiation intensity U is independent of the distance, assuming in both cases that we are in the far field of the antenna (see Sec. 2–13).

2–6 Beam Efficiency

The (total) *beam area* Ω_A (or *beam solid angle*) consists of the main beam area (or solid angle) Ω_M plus the minor-lobe area (or solid angle) Ω_m.[1] Thus,

$$\Omega_A = \Omega_M + \Omega_m \tag{1}$$

The ratio of the main beam area to the (total) beam area is called the (main) *beam efficiency* ε_M. Thus,

$$\textbf{\textit{Beam efficiency}} = \varepsilon_M = \frac{\Omega_M}{\Omega_A} \quad \text{(dimensionless)} \tag{2}$$

The ratio of the minor-lobe area (Ω_m) to the (total) beam area is called the *stray factor*. Thus,

$$\varepsilon_m = \frac{\Omega_m}{\Omega_A} = \text{stray factor} \tag{3}$$

It follows that

$$\varepsilon_M + \varepsilon_m = 1 \tag{4}$$

2–7 Directivity *D* and Gain *G*

The directivity D and the gain G are probably the most important parameters of an antenna.

The *directivity* of an antenna is equal to the ratio of the maximum power density $P(\theta, \phi)_{\max}$ (watts/m^2) to its average value over a sphere as observed in the far field of an antenna. Thus,

[1] If the main beam is not bounded by a deep null, its extent becomes an arbitrary act of judgment.

$$D = \frac{P(\theta, \phi)_{max}}{P(\theta, \phi)_{av}} \qquad \textbf{Directivity from pattern} \tag{1}$$

The directivity is a dimensionless ratio ≥ 1.

The average power density over a sphere is given by

$$\begin{aligned} P(\theta, \phi)_{av} &= \frac{1}{4\pi} \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi} P(\theta, \phi) \sin\theta \, d\theta \, d\phi \\ &= \frac{1}{4\pi} \iint_{4\pi} P(\theta, \phi) \, d\Omega \qquad (\text{W sr}^{-1}) \end{aligned} \tag{2}$$

Therefore, the directivity

$$D = \frac{P(\theta, \phi)_{max}}{(1/4\pi) \iint_{4\pi} P(\theta, \phi) \, d\Omega} = \frac{1}{(1/4\pi) \iint_{4\pi} [P(\theta, \phi)/P(\theta, \phi)_{max}] \, d\Omega} \tag{3}$$

and

$$D = \frac{4\pi}{\iint_{4\pi} P_n(\theta, \phi) \, d\Omega} = \frac{4\pi}{\Omega_A} \qquad \textbf{Directivity from beam area } \Omega_A \tag{4}$$

where $P_n(\theta, \phi) \, d\Omega = P(\theta, \phi)/P(\theta, \phi)_{max}$ = normalized power pattern

Thus, the directivity is the ratio of the area of a sphere (4π sr) to the beam area Ω_A of the antenna (Fig. 2–5b).

The smaller the beam area, the larger the directivity D. For an antenna that radiates over only half a sphere the beam area $\Omega_A = 2\pi$ sr (Fig. 2–6) and the directivity is

$$D = \frac{4\pi}{2\pi} = 2 \qquad (= 3.01 \text{ dBi}) \tag{5}$$

where dBi = decibels over isotropic

Note that the idealized *isotropic antenna* ($\Omega_A = 4\pi$ sr) has the lowest possible directivity $D = 1$. All actual antennas have directivities greater than 1 ($D > 1$). The simple short dipole has a beam area $\Omega_A = 2.67\pi$ sr and a directivity $D = 1.5$ ($= 1.76$ dBi).

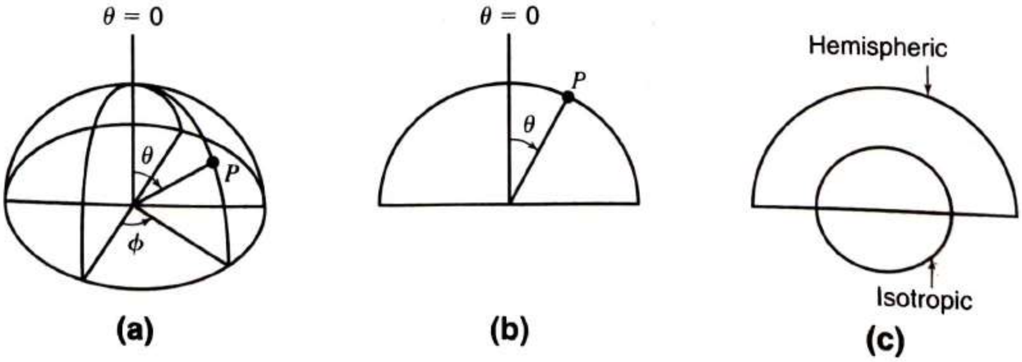


Figure 2–6 Hemispheric power patterns, (a) and (b), and comparison with isotropic pattern (c).

The *gain G* of an antenna is an actual or realized quantity which is less than the directivity D due to ohmic losses in the antenna or its radome (if it is enclosed). In transmitting, these losses involve power fed to the antenna which is not radiated but heats the antenna structure. A mismatch in feeding the antenna can also reduce the gain. The ratio of the gain to the directivity is the *antenna efficiency factor*. Thus,

$$G = kD \tag{6}$$

where k = efficiency factor ($0 \le k \le 1$), dimensionless.

In many well-designed antennas, k may be close to unity. In practice, G is always less than D, with D its maximum idealized value.

Gain can be measured by comparing the maximum power density of the Antenna Under Test (AUT) with a reference antenna of known gain, such as a short dipole. Thus,

$$\text{Gain} = G = \frac{P_{\max}(\text{AUT})}{P_{\max}(\text{ref. ant.})} \times G(\text{ref. ant.}) \tag{7}$$

If the half-power beamwidths of an antenna are known, its directivity

$$D = \frac{41{,}253^{\square}}{\theta^{\circ}_{HP}\phi^{\circ}_{HP}} \tag{8}$$

where

$$\begin{aligned} 41{,}253^{\square} &= \text{number of square degrees in sphere} = 4\pi(180/n)^2 \text{ square degrees } (^{\square}) \\ \theta^{\circ}_{HP} &= \text{half} - \text{power beamwidth in one principal plane} \\ \phi^{\circ}_{HP} &= \text{half} - \text{power beamwidth in other principal plane} \end{aligned}$$

Since (8) neglects minor lobes, a better approximation is a

$$\boxed{D = \frac{40{,}000^{\square}}{\theta^{\circ}_{HP}\phi^{\circ}_{HP}} \qquad \textbf{\textit{Approximate directivity}}} \tag{9}$$

If the antenna has a main half-power beamwidth (HPBW) = 20° in both principal planes, its directivity

$$D = \frac{40{,}000^{\square}}{400^{\square}} = 100 \text{ or } 20 \text{ dBi} \tag{10}$$

which means that the antenna radiates 100 times the power in the direction of the main beam as a nondirectional, isotropic antenna.

The *directivity-beamwidth product* 40,000$^{\square}$ is a rough approximation. For certain types of antennas other values may be more accurate, as discussed in later chapters.

If an antenna has a main lobe with both half-power beamwidths (HPBWs) = 20°, its directivity from (8) is *approximately*

$$D = \frac{4\pi(\text{sr})}{\Omega_A(\text{sr})} \cong \frac{41{,}253(\text{deg}^2)}{\theta^{\circ}_{HP}\phi^{\circ}_{HP}} = \frac{41{,}253(\text{deg}^2)}{20^\circ \times 20^\circ}$$

$$\cong 103 \cong 20 \text{ dBi (dB above isotropic)}$$

which means that the antenna radiates a power in the direction of the main-lobe maximum which is about 100 times as much as would be radiated by a nondirectional (isotropic) antenna for the same power input.

EXAMPLE 2–7.1 Gain of Directional Antenna with Three-Dimensional Field Pattern of Fig. 2–3

The antenna is a lossless end-fire array of 10 isotropic point sources spaced $\lambda/4$ and operating with increased directivity. See Sec. 5–13. The normalized field pattern (see Fig. 2–4*a*) is

$$E_n = \sin\left(\frac{\pi}{2n}\right)\frac{\sin(n\psi/2)}{\sin(\psi/2)} \tag{11}$$

where

$$\psi = d_r(\cos\phi - 1) - \frac{\pi}{n}$$
$$d_r = \pi/2$$
$$n = 10$$

Since the antenna is lossless, gain = directivity.

(*a*) Calculate the gain G.
(*b*) Calculate the gain from the approximate equation (9).
(*c*) What is the difference?

■ Solution

(*a*) From (4)

$$\text{Gain } G = \frac{4\pi}{\iint_{4\pi} P_n(\theta,\phi)\, d\Omega}\, d\theta\, d\phi \tag{12}$$

where $P_n(\theta,\phi)$ = normalized power pattern = $[E_n(\theta,\phi)]^2$.
Introducing the given parameters into (11) and (12),

$$G = 17.8 \text{ or } 12.5 \text{ dB} \qquad \textit{Ans. (a)}$$

(*b*) From (9) and HPBW = 40° in Fig. 2–4,

$$\text{Gain} = \frac{40{,}000^{\square}}{(40^\circ)^2} = 25 \text{ or } 14 \text{ dB} \qquad \textit{Ans. (b)}$$

(*c*) $\Delta G = 25/17.8 = 1.40$ or 1.5 dB   *Ans. (c)*
The difference is mostly due to the large minor lobes of the pattern. Changing the formula to

$$\text{Gain} = \frac{28{,}000^{\square}}{(40^\circ)^2} = 17.5 \text{ or } 12.4 \text{ dB}$$

the gain is much closer to that in (*a*). This approximation is considered more appropriate for end-fire arrays with increased directivity.

EXAMPLE 2–7.2 Directivity

The normalized field pattern of an antenna is given by $E_n = \sin\theta\sin\phi$, where θ = zenith angle (measured from z axis) and ϕ = azimuth angle (measured from x axis) (see figure). E_n has a value only for $0 \le \theta \le \pi$

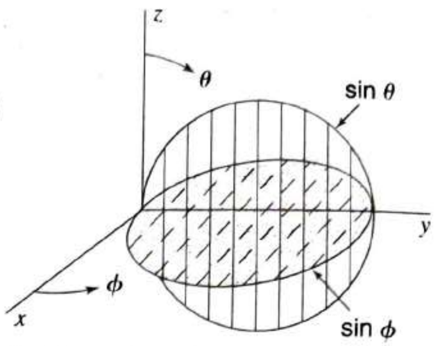
and $0 \le \phi \le \pi$ and is zero elsewhere (pattern is unidirectional with maximum in $+y$ direction). Find (a) the exact directivity, (b) the approximate directivity from (8), and (c) the decibel difference.

■ **Solution**

$$D = \frac{4\pi}{\int_0^{\pi}\int_0^{\pi} \sin^3\theta \sin^2\phi \, d\theta \, d\phi} = \frac{4\pi}{2\pi/3} = 6 \qquad Ans.\ (a)$$

$$D \cong \frac{41{,}253^{\square}}{90^\circ \times 90^\circ} = 5.1 \qquad Ans.\ (b)$$

$$10 \log \frac{6.0}{5.1} = 0.7 \text{ dB} \qquad Ans.\ (c)$$



Unidirectional $\sin\theta$ and $\sin\phi$ field patterns.

2–8 Directivity and Resolution

The resolution of an antenna may be defined as equal to half the beamwidth between first nulls (FNBW)/2,[1] for example, an antenna whose pattern FNBW = 2° has a resolution of 1° and, accordingly, should be able to distinguish between transmitters on two adjacent satellites in the Clarke geostationary orbit separated by 1°. Thus, when the antenna beam maximum is aligned with one satellite, the first null coincides with the adjacent satellite.

Half the beamwidth between first nulls is approximately equal to the half-power beamwidth (HPBW) or

$$\frac{\text{FNBW}}{2} \cong \text{HPBW} \tag{1}$$

Thus, from (2–4–6) the product of the FNBW/2 in the two principal planes of the antenna pattern is a measure of the antenna beam area.[2] Thus,

$$\Omega_A = \left(\frac{\text{FNBW}}{2}\right)_\theta \left(\frac{\text{FNBW}}{2}\right)_\phi \tag{2}$$

It then follows that the number N of radio transmitters or point sources of radiation distributed uniformly over the sky which an antenna can resolve is given approximately by

$$N = \frac{4\pi}{\Omega_A} \tag{3}$$

where Ω_A = beam area, sr

However, from (2–7–4),

$$D = \frac{4\pi}{\Omega_A} \tag{4}$$

and we may conclude that *ideally* the number of point sources an antenna can resolve is numerically equal to the directivity of the antenna or

$$\boxed{D = N} \tag{5}$$

---

[1]Often called the *Rayleigh resolution*. See J. D. Kraus, *Radio Astronomy*, 2d ed., pp. 6–19, Cygnus-Quasar, 1986.

[2]Usually FNBW/2 is slightly greater than the HPBW and (2) is actually a better approximation to Ω_A than $\Omega_A = \theta_{HP}\phi_{HP}$ as given by (2–4–6).

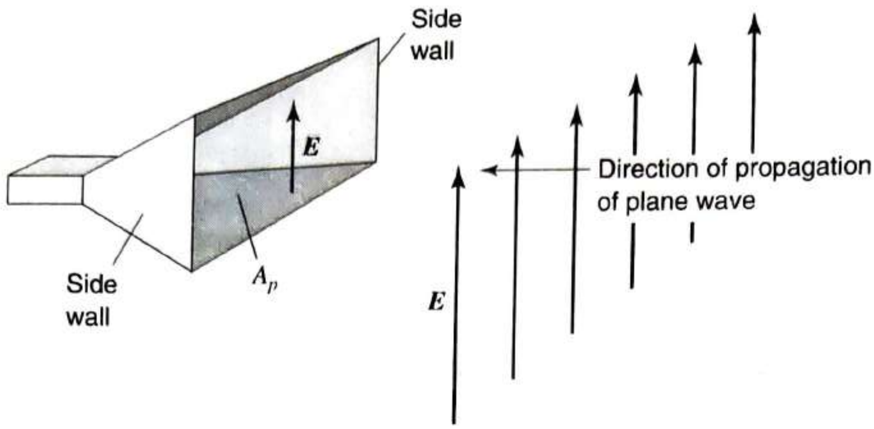


Figure 2–7 Plane wave incident on electromagnetic horn of physical aperture A_p.

Equation (4) states that the directivity is equal to the number of beam areas into which the antenna pattern can subdivide the sky and (5) gives the added significance that the ***directivity is equal to the number of point sources in the sky that the antenna can resolve*** under the assumed ideal conditions of a uniform source distribution.[1]

2–9 Antenna Apertures

The concept of aperture is most simply introduced by considering a receiving antenna. Suppose that the receiving antenna is a rectangular electromagnetic horn immersed in the field of a uniform plane wave as suggested in Fig. 2–7. Let the Poynting vector, or power density, of the plane wave be S watts per square meter and the area, or physical aperture of the horn, be A_p square meters. If the horn extracts all the power from the wave over its entire physical aperture, then the total power P absorbed from the wave is

$$P = \frac{E^2}{Z} A_p = S A_p \qquad \text{(W)} \tag{1}$$

Thus, the electromagnetic horn may be regarded as having an aperture, the total power it extracts from a passing wave being proportional to the aperture or area of its mouth.

But the field response of the horn is NOT uniform across the aperture A because E at the sidewalls must equal zero. Thus, the *effective aperture* A_e of the horn is less than the *physical aperture* A_p as given by

$$\boxed{\varepsilon_{ap} = \frac{A_e}{A_p} \qquad \text{(dimensionless)} \qquad \textit{Aperture efficiency}} \tag{2}$$

where ε_{ap} = aperture efficiency.

For horn and parabolic reflector antenna, aperture efficiencies are commonly in the range of 50 to 80% ($0.5 \le \varepsilon_{ap} \le 0.8$). Large dipole or patch arrays with uniform field to the edges of the physical aperture may attain higher aperture efficiencies approaching 100%. However, to reduce sidelobes, fields are commonly tapered toward the edges, resulting in reduced aperture efficiency.

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[1] A strictly regular distribution of points on a sphere is only possible for 4, 6, 8, 12, and 20 points corresponding to the vertices of a tetrahedron, cube, octahedron, icosahedron and dodecahedron.

Consider now an antenna with an *effective aperture* A_e, which radiates all of its power in a conical pattern of beam area Ω_A, as suggested in Fig. 2–8. Assuming a uniform field E_a over the aperture, the power radiated is

$$P = \frac{E_a^2}{Z_0} A_e \qquad \text{(W)} \tag{3}$$

where Z_0 = intrinsic impedance of medium (377 Ω for air or vacuum).

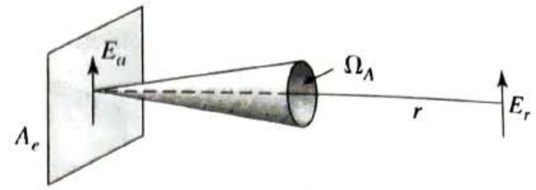


Figure 2–8 Radiation over beam area Ω_A from aperture A_e.

Assuming a uniform field E_r in the far field at a distance r, the power radiated is also given by

$$P = \frac{E_r^2}{Z_0} r^2 \Omega_A \qquad \text{(W)} \tag{4}$$

Equating (3) and (4) and noting that $E_r = E_a A_e / r\lambda$ yields the aperture–beam-area relation

$$\lambda^2 = A_e \Omega_A \qquad (\text{m}^2) \qquad \textit{Aperture–beam-area relation} \tag{5}$$

where Ω_A = beam area (sr).

Thus, if A_e is known, we can determine Ω_A (or vice versa) at a given wavelength. From (5) and (2–7–4) it follows that the directivity

$$D = 4\pi \frac{A_e}{\lambda^2} \qquad \textit{Directivity from aperture} \tag{6}$$

All antennas have an effective aperture which can be calculated or measured. Even the hypothetical, idealized isotropic antenna, for which $D = 1$, has an effective aperture

$$A_e = \frac{D\lambda^2}{4\pi} = \frac{\lambda^2}{4\pi} = 0.0796\lambda^2 \tag{7}$$

All lossless antennas must have an effective aperture equal to or greater than this. By reciprocity the effective aperture of an antenna is the same for receiving and transmitting.

Three expressions have now been given for the *directivity D*. They are

$$D = \frac{P(\theta, \phi)_{\text{max}}}{P(\theta, \phi)_{\text{av}}} \qquad \text{(dimensionless)} \qquad \textit{Directivity from pattern} \tag{8}$$

$$D = \frac{4\pi}{\Omega_A} \qquad \text{(dimensionless)} \qquad \textit{Directivity from pattern} \tag{9}$$

$$D = 4\pi \frac{A_e}{\lambda^2} \qquad \text{(dimensionless)} \qquad \textit{Directivity from aperture} \tag{10}$$

When the antenna is receiving with a load resistance R_L matched to the antenna radiation resistance R_r ($R_L = R_r$), as much power is reradiated from the antenna as is delivered to the load. This is the condition of *maximum power transfer* (antenna assumed lossless).

In the circuit case of a load matched to a generator, as much power is dissipated in the generator as is delivered to the load. Thus, for the case of the dipole antenna in Fig. 2–9 we have a *load power*

$$P_{\text{load}} = S A_e \qquad \text{(W)} \tag{11}$$

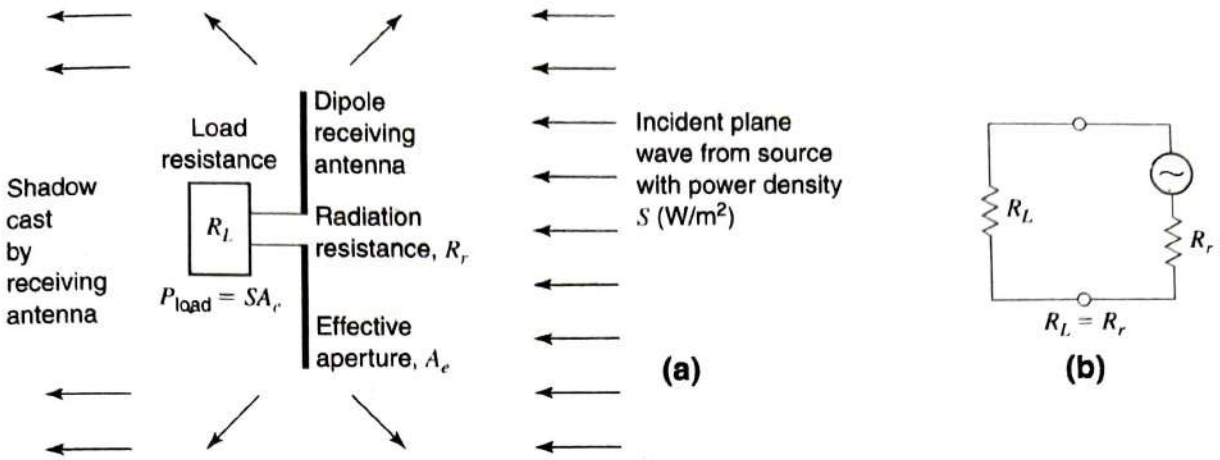


Figure 2-9 (*a*) The receiving antenna matched to a load ($R_r = R_L$) reradiates a power that is equal to the power delivered to the load. More generally, the reradiated and scattered power from any antenna or object yields a *radar cross-section* (RCS) which is proportional to the back-scattered power received at a radar at a distance r, as discussed in Chapter 12. (*b*) Equivalent circuit.

where

$$S = \text{power density at receiving antenna, W/m}^2$$
$$A_e = \text{effective aperture of antenna, m}^2$$

and a *reradiated power*

$$P_{rerad} = \frac{\text{Power reradiated}}{4\pi \text{ sr}} = SA_r \qquad \text{(W)}$$

where A_r = reradiating aperture = A_e, m² and

$$P_{rerad} = P_{load}$$

The above discussion is applicable to a single dipole ($\lambda/2$ or shorter). However, it does not apply to all antennas. In addition to the reradiated power, an antenna may scatter power that does not enter the antenna-load circuit. Thus, the reradiated plus scattered power may exceed the power delivered to the load. See Sec. 21–15 for a discussion that includes both receiving and transmitting conditions.

2–10 Effective Height

The *effective height h* (meters) of an antenna is another parameter related to the aperture. Multiplying the effective height by the incident field E (volts per meter) of the same polarization gives the voltage V induced. Thus,

$$V = hE \tag{1}$$

Accordingly, the effective height may be defined as the ratio of the induced voltage to the incident field or

$$h = \frac{V}{E} \qquad \text{(m)} \tag{2}$$

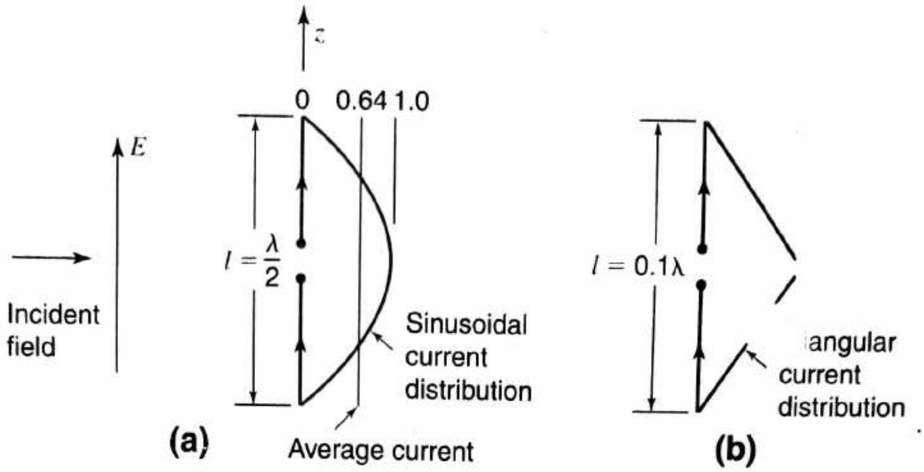


Figure 2–9–1 (*a*) Dipole of length $l = \lambda/2$ with sinusoidal current distribution. (*b*) Dipole of length $l = 0.1\lambda$ with triangular current distribution.

Consider, for example, a vertical dipole of length $l = \lambda/2$ immersed in an incident field E, as in Fig. 2–9–1(a). If the current distribution of the dipole were uniform, its effective height would be l. The actual current distribution, however, is nearly sinusoidal with an *average value* $2/\pi = 0.64$ (of the maximum) so that its effective height $h = 0.64l$. It is assumed that the antenna is oriented for maximum response.

If the same dipole is used at a longer wavelength so that it is only 0.1λ long, the current tapers almost linearly from the central feed point to zero at the ends in a triangular distribution, as in Fig. 2–9–1(b). The average current is 1/2 of the maximum so that the effective height is $0.5l$.

Thus, another way of defining effective height is to consider the transmitting case and equate the effective height to the physical height (or length l) multiplied by the (normalized) average current or

$$h_e = \frac{1}{I_0}\int_0^{h_p} I(z)\,dz = \frac{I_{av}}{I_0}h_p \qquad \text{(m)} \tag{3}$$

where

h_e = effective height, m
h_p = physical height, m
I_{av} = average current, A

It is apparent that *effective height* is a useful parameter for transmitting tower-type antennas.[1] It also has an application for small antennas. The parameter *effective aperture* has more general application to all types of antennas. The two have a simple relation, as will be shown.

[1]Effective height can also be expressed more generally as a vector quantity. Thus (for linear polarization) we can write

$$V = \mathbf{h}_e \cdot \mathbf{E} = h_e E \cos\theta$$

where

$\mathbf{h}_e$ = effective height and polarization angle of antenna, m
$\mathbf{E}$ = field intensity and polarization angle of incident wave, V m^{-1}
θ = angle between polarization angles of antenna and wave, deg

In a still more general expression (for any polarization state), θ is the angle between polarization states on the Poincaré sphere (see Sec. 2–17).

For an antenna of radiation resistance R_r matched to its load, the power delivered to the load is equal to

$$P = \frac{1}{4}\frac{V^2}{R_r} = \frac{h^2 E^2}{4R_r} \qquad \text{(W)} \tag{4}$$

In terms of the effective aperture the same power is given by

$$P = SA_e = \frac{E^2 A_e}{Z_0} \qquad \text{(W)} \tag{5}$$

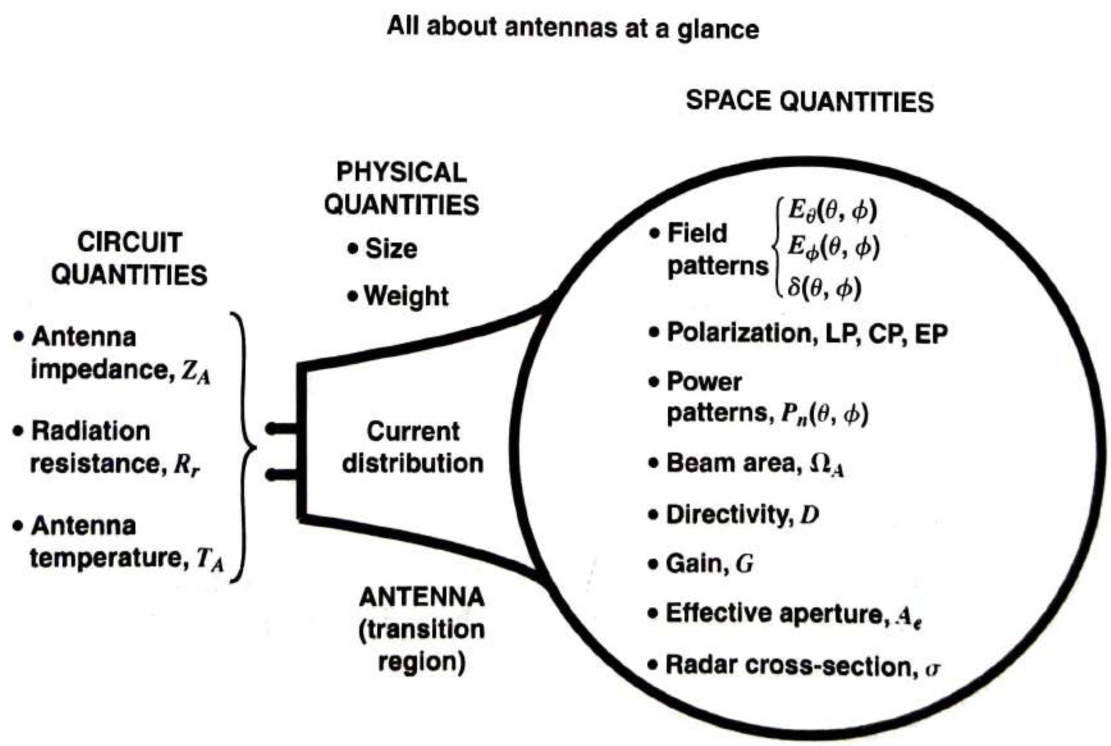
where Z_0 = intrinsic impedance of space (= 377 Ω)

Equating (4) and (5), we obtain

$$h_e = 2\sqrt{\frac{R_r A_e}{Z_0}} \quad \text{(m)} \qquad \text{and} \qquad A_e = \frac{h_e^2 Z_0}{4R_r} \quad (\text{m}^2) \tag{6}$$

Thus, effective height and effective aperture are related via radiation resistance and the intrinsic impedance of space.

To summarize, we have discussed the space parameters of an antenna, namely, field and power patterns, beam area, directivity, gain, and various apertures. We have also discussed the circuit quantity of radiation resistance and alluded to antenna temperature, which is discussed further in Sec. 17–1. Figure 2–10 illustrates this ***duality*** of an antenna.



EXAMPLE 2–10.1 Effective Aperture and Directivity of a Short Dipole Antenna

A plane wave is incident on a short dipole as in Fig. 2–11. The wave is assumed to be linearly polarized with E in the y direction. The current on the dipole is assumed constant and in the same phase over its entire length, and the terminating resistance R_T is assumed equal to the dipole radiation resistance R_r. The antenna loss resistance R_L is assumed equal to zero. What is (*a*) the dipole's maximum effective aperture and (*b*) its directivity?

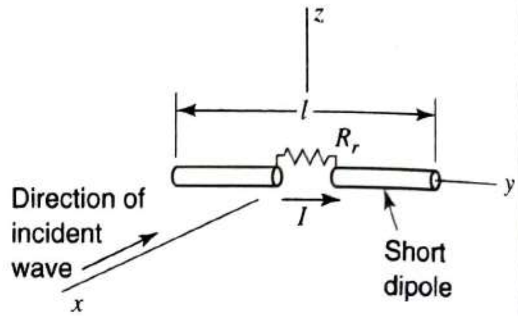


Figure 2–11 Short dipole with uniform current induced by incident wave.

■ Solution

(*a*) The maximum effective aperture of an antenna is

$$A_{em} = \frac{V^2}{4SR_r} \tag{7}$$

where the effective value of the induced voltage V is here given by the product of the effective electric field intensity at the dipole and its length, that is,

$$V = El \tag{8}$$

The radiation resistance R_r of a short dipole of length l with uniform current will be shown later to be

$$R_r = \frac{80\pi^2 l^2}{\lambda^2}\left(\frac{I_{av}}{I_0}\right)^2 = 790\left(\frac{I_{av}}{I_0}\right)^2\left(\frac{l}{\lambda}\right)^2 \quad (\Omega) \tag{9}$$

where

λ = wavelength

I_{av} = average current

I_0 = terminal current

The power density, or Poynting vector, of the incident wave at the dipole is related to the field intensity by

$$S = \frac{E^2}{Z} \tag{10}$$

where Z = intrinsic impedance of the medium.

In the present case, the medium is free space so that $Z = 120\pi\ \Omega$. Now substituting (8), (9), and (10) into (7), we obtain for the maximum effective aperture of a short dipole (for $I_{av} = I_0$)

$$A_{em} = \frac{120\pi E^2 l^2 \lambda^2}{320\pi^2 E^2 l^2} = \frac{3}{8\pi}\lambda^2 = 0.119\lambda^2 \qquad Ans.\ (a)$$

(*b*) $$D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \times 0.119\lambda^2}{\lambda^2} = 1.5 \qquad Ans.\ (b)$$

A typical short dipole might be $\lambda/10$ long and $\lambda/100$ in diameter for a physical cross-sectional aperture of $0.001\lambda^2$ as compared to the $0.119\lambda^2$ effective aperture of Example 2–10.1. Thus, a single dipole or linear antenna may have a physical aperture that is smaller than its effective aperture. However, a *broadside array* of many dipoles or linear antennas has an *overall* physical aperture that, like horns and dishes, is larger than its effective aperture. On the other hand, an end-fire array of dipoles, as in a Yagi-Uda antenna, has an *end-on* physical cross section that is smaller than the antenna's effective aperture. Thus, depending on the antenna, physical apertures may be larger than effective apertures, or vice versa.

EXAMPLE 2–10.2 Effective Aperture and Directivity of Linear $\lambda/2$ Dipole

A plane wave incident on the antenna is traveling in the negative x direction as in Fig. 2–12*a*. The wave is linearly polarized with E in the y direction. The equivalent circuit is shown in Fig. 2–12*b*. The antenna has been replaced by an equivalent or Thévenin generator. The infinitesimal voltage dV of this generator due to the voltage induced by the incident wave in an infinitesimal element of length dy of the antenna is

$$dV = E\,dy \cos\frac{2\pi y}{\lambda} \tag{11}$$

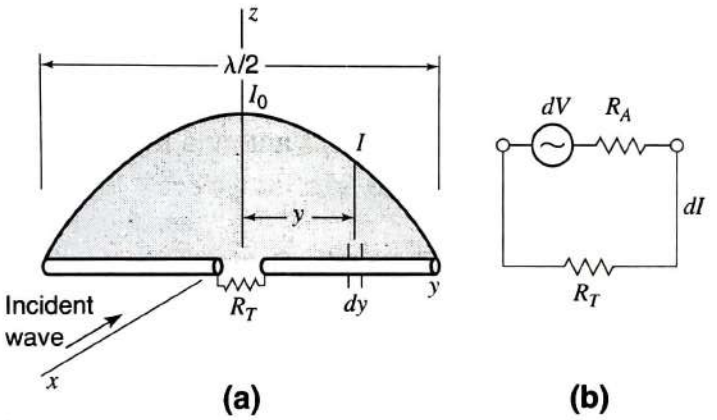


Figure 2–12 Linear $\lambda/2$ antenna in field of electromagnetic wave (*a*) and equivalent circuit (*b*).

It is assumed that the infinitesimal induced voltage is proportional to the current at the infinitesimal element as given by the current distribution (11). Find (*a*) the effective aperture and (*b*) the directivity of the $\lambda/2$ dipole.

■ **Solution**

(*a*) The total induced voltage V is given by integrating (11) over the length of the antenna. This may be written as

$$V = 2\int_0^{\lambda/4} E \cos\frac{2\pi y}{\lambda}\,dy \tag{12}$$

Performing the integration in (12) we have

$$V = \frac{E\lambda}{\pi} \tag{13}$$

The value of the radiation resistance R_r of the linear $\lambda/2$ antenna will be taken as 73 Ω. The terminating resistance R_T is assumed equal to R_r.

Thus, we obtain, for the maximum effective aperture of a linear $\lambda/2$ antenna,

$$A_{em} = \frac{120\pi E^2\lambda^2}{4\pi^2 E^2 \times 73} = \frac{30}{73\pi}\lambda^2 = 0.13\lambda^2 \qquad \textit{Ans. (a)}$$

$$(b)\ D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \times 0.13\lambda^2}{\lambda^2} = 1.63 \qquad \textit{Ans. (b)}$$

The maximum effective aperture of the linear $\lambda/2$ antenna is about 10 percent greater than that of the short dipole.

The maximum effective aperture of the $\lambda/2$ antenna is approximately the same as an area 1/2 by $1/4\lambda$ on a side, as illustrated in Fig. 2–13*a*. This area is $0.125\lambda^2$. An elliptically shaped aperture of $0.13\lambda^2$ is shown in Fig. 2–13*b*. The physical significance of these apertures is that power from the incident plane wave is absorbed over an area of this size and is delivered to the terminal resistance or load.

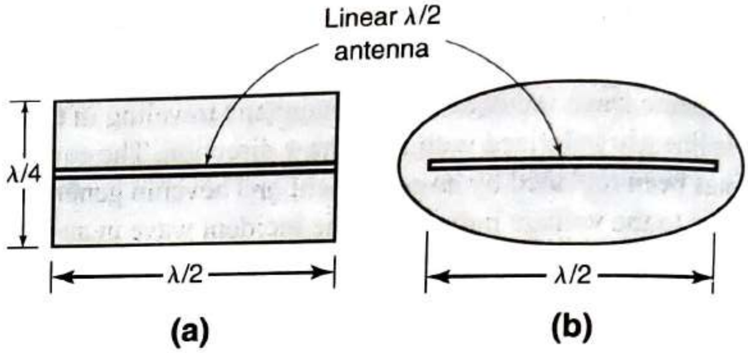


Figure 2–13 (*a*) Maximum effective aperture of linear $\lambda/2$ antenna is approximately represented by rectangle 1/2 by $1/4\lambda$ on a side. (*b*) Maximum effective aperture of linear $\lambda/2$ antenna represented by elliptical area of $0.13\lambda^2$.

Although the radiation resistance, effective aperture, and directivity are the same for both receiving and transmitting, the current distribution is, in general, not the same. Thus, a plane wave incident on a receiving antenna excites a different current distribution than a localized voltage applied to a pair of terminals for transmitting.

2–11 The Radio Communication Link

The usefulness of the aperture concept is well illustrated by using it to derive the important Friis transmission formula published in 1946 by Harald T. Friis (1) of the Bell Telephone Laboratory.

Referring to Fig. 2–14, the formula gives the power received over a radio communication link. Assuming lossless, matched antennas, let the transmitter feed a power P_t to a transmitting antenna of effective aperture A_{et}. At a distance r a receiving antenna of effective aperture A_{er} intercepts some of the power radiated by the transmitting antenna and delivers it to the receiver R. Assuming for the moment that the transmitting antenna is isotropic, the power per unit area available at the receiving antenna is

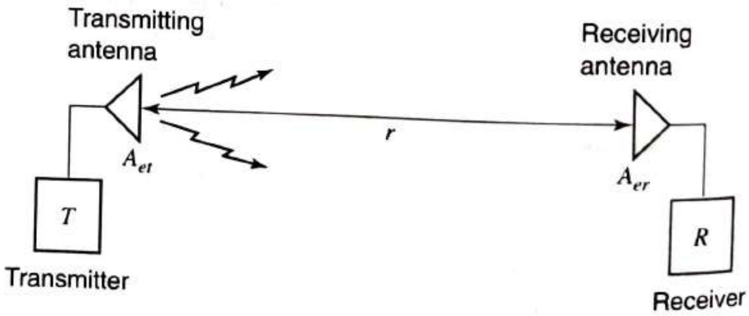


Figure 2–14 Communication circuit with waves from transmitting antenna arriving at the receiving antenna by a direct path of length r.

$$S_r = \frac{P_t}{4\pi r^2} \qquad \text{(W)} \qquad (1)$$

If the antenna has gain G_t, the power per unit area available at the receiving antenna will be increased in proportion as given by

$$S_r = \frac{P_t G_t}{4\pi r^2} \qquad \text{(W)} \tag{2}$$

Now the power collected by the lossless, matched receiving antenna of effective aperture A_{er} is

$$P_r = S_r A_{er} = \frac{P_t G_t A_{er}}{4\pi r^2} \qquad \text{(W)} \tag{3}$$

The gain of the transmitting antenna can be expressed as

$$G_t = \frac{4\pi A_{et}}{\lambda^2} \tag{4}$$

Substituting this in (3) yields the *Friis transmission formula*

$$\boxed{\frac{P_r}{P_t} = \frac{A_{er} A_{et}}{r^2 \lambda^2} \qquad \text{(dimensionless)} \qquad \textbf{\textit{Friis transmission formula}}} \tag{5}$$

where

P_r = received power, W
P_t = transmitted power, W
A_{et} = effective aperture of transmitting antenna, m^2
A_{er} = effective aperture of receiving antenna, m^2
r = distance between antennas, m
λ = wavelength, m

EXAMPLE 2–11.1 Radio Communication Link

A radio link has a 15-W transmitter connected to an antenna of 2.5 m^2 effective aperture at 5 GHz. The receiving antenna has an effective aperture of 0.5 m^2 and is located at a 15-km line-of-sight distance from the transmitting antenna. Assuming lossless, matched antennas, find the power delivered to the receiver.

■ **Solution**

From (5)

$$P = P_t \frac{A_{et} A_{er}}{r^2 \lambda^2} = 15 \frac{2.5 \times 0.5}{15^2 \times 10^6 \times 0.06^2} = 23\ \mu\text{W} \qquad \textit{Ans.}$$

2–12 Fields from Oscillating Dipole

Although a charge moving with uniform velocity along a straight conductor does not radiate, a charge moving back and forth in simple harmonic motion along the conductor is subject to acceleration (and deceleration) and radiates.

To illustrate radiation from a dipole antenna, let us consider that the dipole of Fig. 2–15 has two equal charges of opposite sign oscillating up and down in harmonic motion with instantaneous separation l (maximum separation l_0) while focusing attention on the electric field. For clarity only a single electric field line is shown.

At time $t = 0$ the charges are at maximum separation and undergo maximum acceleration $\dot{v}$ as they reverse direction (Fig. 2–15a). At this instant the current I is zero. At an $\frac{1}{8}$-period later, the charges are moving toward

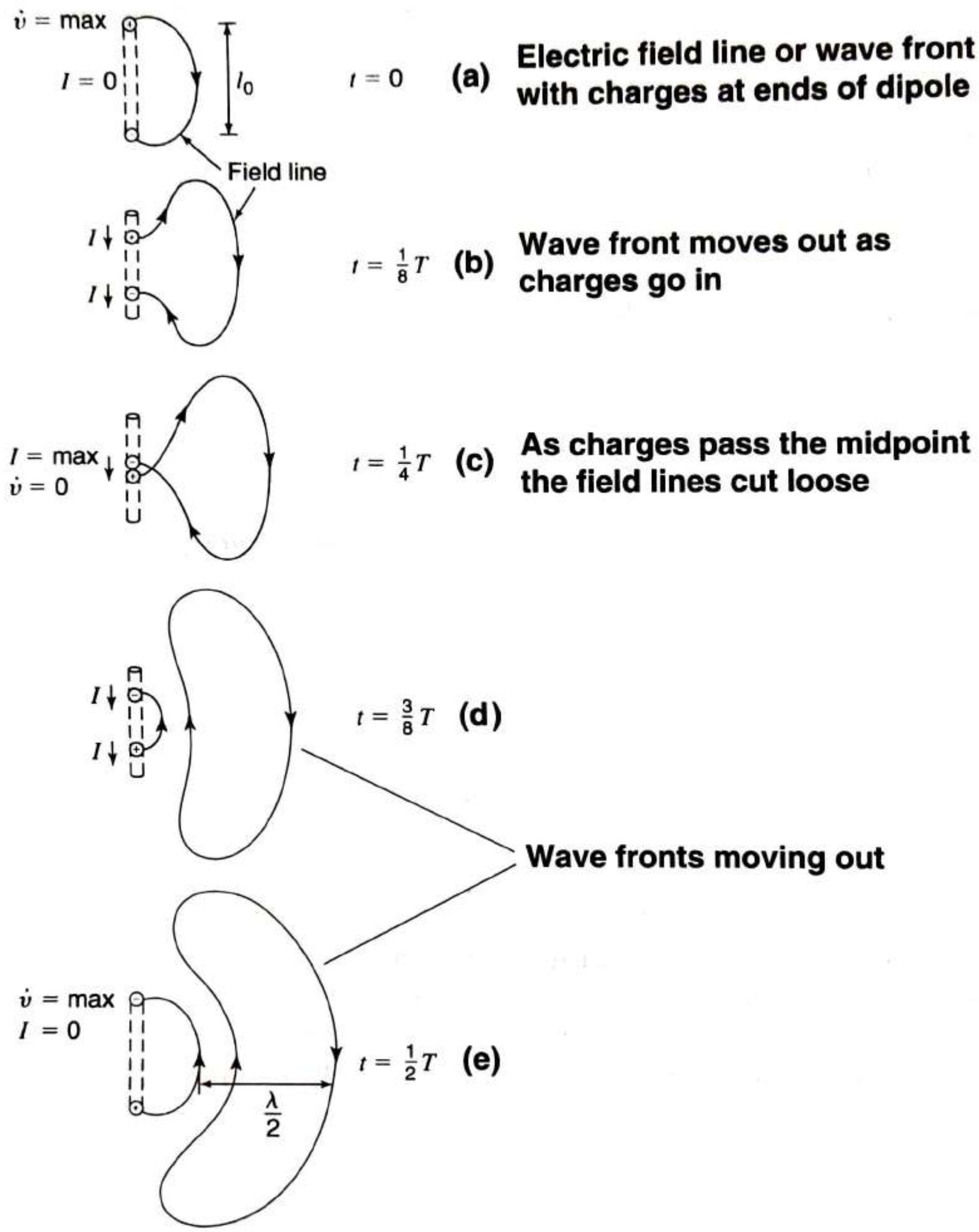


Figure 2–15 Oscillating electric dipole consisting of two electric charges in simple harmonic motion, showing propagation of an electric field line and its detachment (radiation) from the dipole. Arrows next to the dipole indicate current (I) direction.

each other (Fig. 2–15*b*) and at a $\frac{1}{4}$-period they pass at the midpoint (Fig. 2–15*c*). As this happens, the field lines detach and new ones of opposite sign are formed. At this time the equivalent current I is a maximum and the charge acceleration is zero. As time progresses to a $\frac{1}{2}$-period, the fields continue to move out as in Fig. 2–15*d* and *e*.

An oscillating dipole with more feed lines is shown in Fig. 2–16 at five instants of time. In order to understand the process of radiation shown in Fig. 2–16, imagine that a smoker is displaying his capability of making rings of smoke. As the rings move farther, their size increases. With the assumed maintenance of shape, a distant ring will surely be of bigger size with lesser smoke density.

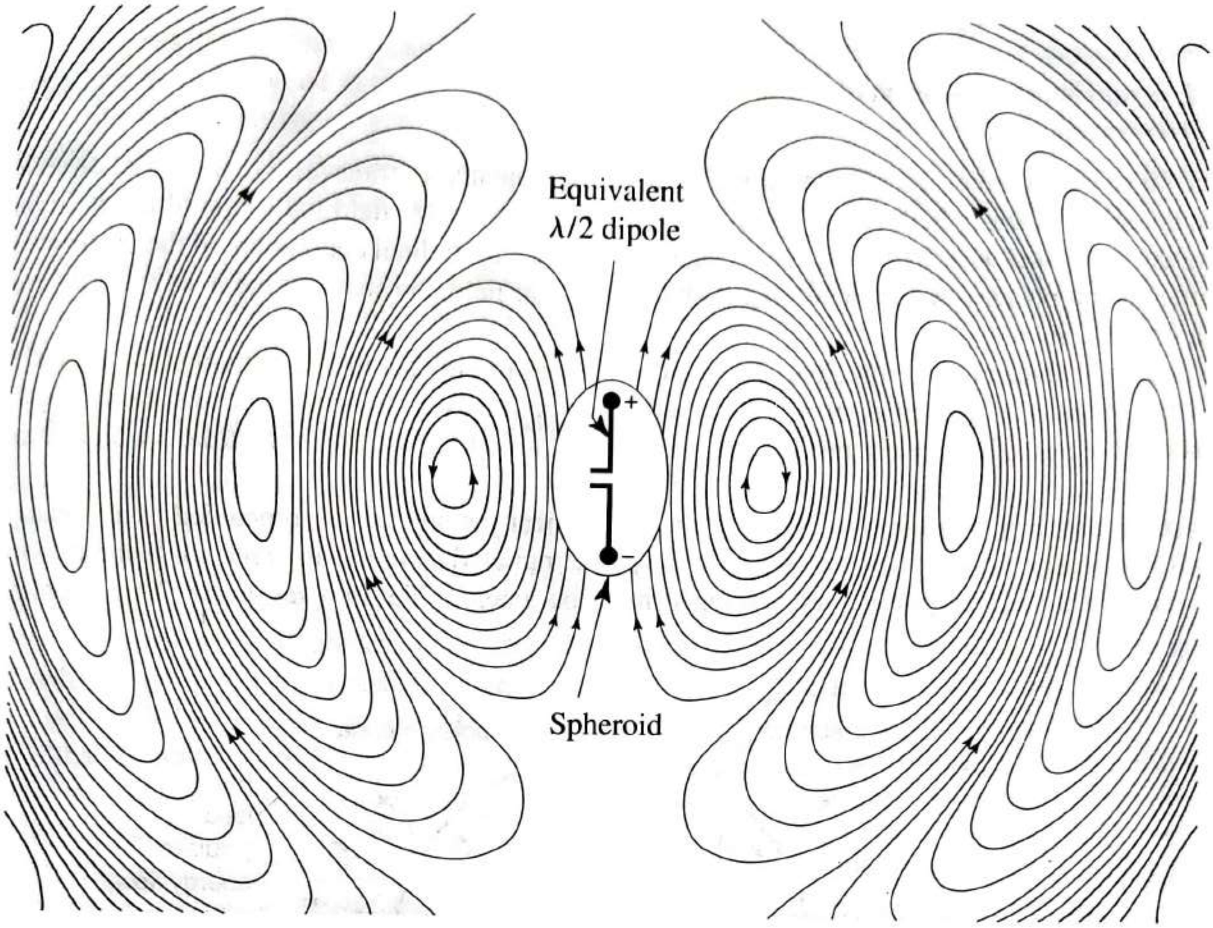


Figure 2–16 Electric field lines of the radiation moving out from $\lambda/2$ dipole antenna. (*Produced by Edward M. Kennaugh, courtesy of John D. Cowan, Jr.*)

This analogy conforms to distribution of the electric field intensity at farther and farther distances from the source as has been illustrated by Fig. 2–16. The concept of direction and separation of lines shown, however, shall not be applicable to the smoke.

2–13 Antenna Field Zones

The fields around an antenna may be divided into two principal regions, one near the antenna called the *near field* or *Fresnel zone* and one at a large distance called the *far field* or *Fraunhofer zone*. Referring to Fig. 2–17, the boundary between the two may be arbitrarily taken to be at a radius

$$R = \frac{2L^2}{\lambda} \quad \text{(m)} \tag{1}$$

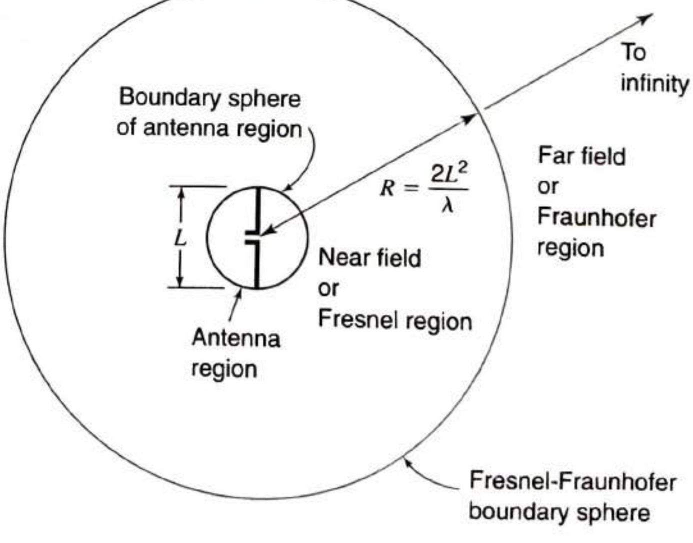


Figure 2–17 Antenna region, Fresnel region and Fraunhofer region.

where

L = maximum dimension of the antenna, m

λ = wavelength, m

In the far or Fraunhofer region, the measurable field components are transverse to the radial direction from the antenna and all power flow is directed radially outward. In the far field the shape of the field pattern is independent of the distance. In the near or Fresnel region, the longitudinal component of the electric field may be significant and power flow is not entirely radial. In the near field, the shape of the field pattern depends, in general, on the distance.

Enclosing the antenna in an imaginary boundary sphere as in Fig. 2–18a it is as though the region near the poles of the sphere acts as a reflector. On the other hand, the waves expanding perpendicular to the dipole in the equatorial region of the sphere result in power leakage through the sphere as if partially transparent in this region.

This results in reciprocating (oscillating) energy flow near the antenna accompanied by outward flow in the equatorial region. The outflow accounts for the power radiated from the antenna, while the reciprocating energy represents reactive power that is trapped near the antenna like in a resonator. This oversimplified

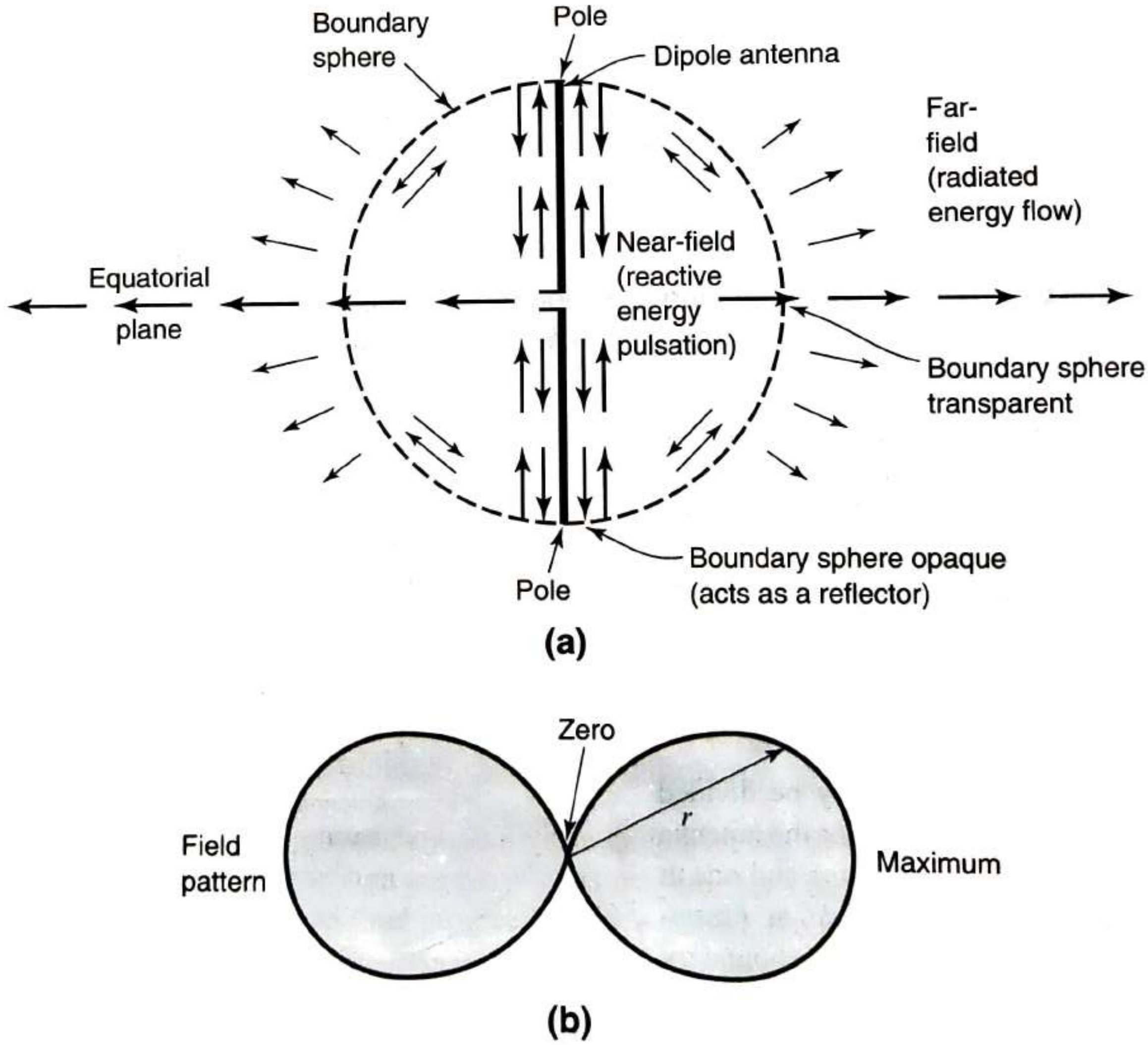


Figure 2–18 Energy flow near a dipole antenna (a) and radiation field pattern (b). The radius vector r is proportional to the field radiated in that direction.

discussion accounts in a qualitative way for the field pattern of the $\lambda/2$ dipole antenna as shown in Fig. 2–18*b*. The energy picture is discussed in more detail in Sec. 6.3 and displayed in Fig. 6–6.

For a $\lambda/2$ dipole antenna, the energy is stored at one instant of time in the electric field, mainly near the ends of the antenna or maximum charge regions, while a $\frac{1}{2}$-period later the energy is stored in the magnetic field mainly near the center of the antenna or maximum current region.

Note that although the term *power flow* is sometimes used, it is actually *energy* which flows, power being the time rate of energy flow. A similar loose usage occurs when we say we pay a power bill, when, in fact, we are actually paying for electric energy.

2–14 Shape-impedance Considerations

It is possible in many cases to deduce the qualitative behavior of an antenna from its shape. This may be illustrated with the aid of Fig. 2–19. Starting with the opened-out two-conductor transmission line of Fig. 2–19*a*, we find that, if extended far enough, a nearly constant impedance will be provided at the input (left) end for $d \ll \lambda$ and $D \geq \lambda$.

In Fig. 2–19*b* the curved conductors are straightened into regular cones and in Fig. 2–19*c* the cones are aligned collinearly, forming a biconical antenna. In Fig. 2–19*d* the cones degenerate into straight wires. In going from Fig. 2–19*a* to *d*, the bandwidth of relatively constant impedance tends to decrease. Another difference is that the antennas of Fig. 2–19*a* and *b* are unidirectional with beams to the right, while the antennas of Fig. 2–19*c* and *d* are omnidirectional in the horizontal plane (perpendicular to the wire or cone axes).

A different modification is shown in Fig. 2–19*e*. Here the two conductors are curved more sharply and in opposite directions, resulting in a spiral antenna with maximum radiation broadside (perpendicular to the page) and with polarization which rotates clockwise. This antenna, like the one in Fig. 2–19*a*, exhibits very broadband characteristics (see Chap. 18).

The dipole antennas of Fig. 2–19 are balanced, i.e., they are fed by two-conductor (balanced) transmission lines. Figure 2–20 illustrates a similar evolution of monopole antennas, i.e., antennas fed from coaxial (unbalanced) transmission lines.

By gradually tapering the inner and outer conductors or a coaxial transmission line, a very wide band antenna with an appearance reminiscent of a volcanic crater and puff of smoke is obtained, as suggested in the cutaway view of Fig. 2–20*a*.

In Fig. 2–20*b* the volcano form is modified into a double dish and in Fig. 2–20*c* into two wide-angle cones. All of these antennas are omnidirectional in a plane perpendicular to their axes and all have a wide bandwidth. For example, an actual biconical antenna, as in Fig. 2–20*c*, with a full cone angle of 120° has an omnidirectional pattern and nearly constant 50-Ω input impedance (power reflection less than 1 percent or VSWR < 1.2) over a 6 to 1 bandwidth with cone diameter $D = \lambda$ at the lowest frequency.

Increasing the lower cone angle to 180° or into a flat ground plane while reducing the upper cone angle results in the antenna of Fig. 2–20*d*. Collapsing the upper cone into a thin stub, we arrive at the extreme modification of Fig. 2–20*e*. If the antenna of Fig. 2–20*a* is regarded as the most basic form, the stub type of Fig. 2–20*e* is the most degenerate form, with a relatively narrow bandwidth.

> Antennas with large and abrupt discontinuities have large reflections and act as reflectionless transducers only over narrow frequency bands where the reflections cancel. Antennas with discontinuities that are small and gradual have small reflections and are, in general, relatively reflectionless transducers over wide frequency bands.